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Completion of the Diffusion Wave Flood Tracking Model Using the Method of Lines (MOL)

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Abstract

The flood-tracking model is a method that can be used to predict when a flood will occur. The flood wave model is developed using a diffusion equation consisting of mass conservation equations and momentum conservation equations. This research was conducted to determine the application of the Method of Lines (MOL) in solving flood tracking models using the diffusion equation. The steps involved are discretizing the flood wave diffusion tracking equation by replacing the spatial derivative (x) using the central difference method, resulting in a system of ordinary differential equations. Then, solving the system of ordinary differential equations using the fourth-order Runge Kutta method. The approach used in this research is quantitative. Simulations are performed by inputting a sample case and entering the data into the MATLAB program. The flow discharge produced increases as the flow velocity increases, and the resulting graph becomes more concave as the velocity increases. Thus, by knowing the changes in flow velocity, flow width, and flow depth in the upstream area of the river, it can be predicted how much the water discharge will change at each observation point downstream of the river.

1. INTRODUCTION

The flood-tracking model has long been developed by the U.S. Army Corps of Engineers and Mc Carthy on the Muskingum River in 1934 to 1935, which is known as the Muskingum method. Then, this method was developed again by Cunge in 1980 using finite difference theory [1]. The flood tracking model can be used to predict when a flood will occur, so that it can reduce the impact of flood disasters such as material, economic damage, and even loss of life [2]. According to [3], the flood tracking model has 3 approaches, including the kinematic wave model, the dynamic wave model and the diffusion wave model. The diffusion wave flood tracking model is influenced by the average flow velocity and the slope of the channel bed [4]. This model has two forms, namely the conservative form and the non-conservative form. According to [3] a conservative diffusion wave flood tracking model was used to obtain flood flow discharge. And a non-conservative diffusion wave flood tracking model is used to obtain the height of the flood water level. Several previous researchers have conducted research on flood tracking models using numerical methods to determine predictions of flood occurrence. For example, research discussing mathematical modeling for diffusion wave flood tracking in a conservative form was carried out by [5], then a flood tracking model was developed by [6] using the Duffort Frankel method.

To solve a wave equation can be done using two methods, namely analytically and numerically. Because analytical solutions are not always easy to obtain solutions or even solutions cannot be obtained, numerical solutions are a method that can be used to solve all problems in differential equations. However, there are several numerical methods used that tend to be computationally unstable, so stability tests are necessary. One of them is using von Neumann stability analysis. According to [7], von Neumann stability is a method that can be used to find out how stable or unstable a numerical scheme is. In von Neumann stability, a numerical scheme is said to be stable if $|B|\leq 1$.

The conservative form of diffusion wave flood tracking model can be solved using numerical methods, one of which uses the Method of Lines (MOL). The MOL was first introduced by Erich Rohe in 1930, who was a mathematician from Germany. This method is one of the most efficient numerical methods for solving a Partial Differential Equation [8]. The basic idea of the MOL is carried out by using an algebraic approach to change all the derivatives in the Partial Differential Equation and only one independent variable remains, namely the time variable. Then apply the initial value problem solving method to obtain a numerical solution [9]. The MOL provides a simple, precise computational procedure, and the resulting solution is increasingly accurate [10]. The advantage of the MOL in completing this conservative form of diffusion wave flood tracing model is that there are no researchers who have completed the conservative form of diffusion wave flood tracing model using the MOL. So, research needs to be carried out to find out whether a conservative form of diffusion wave flood tracking model can be solved using the MOL.

2. RESEARCH METHOD

This research uses secondary data obtained from research conducted by [5]. The data includes flow velocity, length of flow section, width of flow, depth of flow, upstream and downstream boundary conditions, initial discharge, Chezy coefficient, number of pias in the flow section, time interval, distance interval, flood wave speed, cross-sectional area, radius hydraulic and diffusion coefficient. Subsequently, various data will be varied for comparison, including flow speed, flow width and flow depth.

3. RESULT AND DISCUSSION

In 2010, Novak, et al., derived the equation of the diffusion wave flood-tracking model [6] and obtain the following equation:

$$\frac{\partial Q}{\partial t} + \lambda \frac{\partial Q}{\partial x} - D \frac{\partial^2 Q}{\partial x^2} = 0 \tag{1}$$

3.1 Discretization of Models with Method of Lines (MOL)

The discretization process of the model is carried out by changing the derivative over space in the partial differential equation using a central finite difference approach. The central finite difference approximation for the first derivative of a space variable is as follows:

$$\frac{\partial Q}{\partial x} = \frac{Q_{i+1}^n - Q_{i-1}^n}{2\Delta x} \tag{2}$$

and for the second derivative of the variable with respect to space:

$$\frac{\partial^2 Q}{\partial x^2} = \frac{Q_{i+1}^n - 2Q_i^n + Q_{i-1}^n}{\Delta x^2} \tag{3}$$

by substituting the approximation in Equations (2) and (3) into Equation (1). Furthermore, derivatives over space are no longer expressed explicitly in terms of space-free variables. Thus, we obtain:

$$\frac{dQ_i^n}{dt} + \lambda \left(\frac{Q_{i+1}^n - Q_{i-1}^n}{2\Delta x} \right) - D \left(\frac{Q_{i+1}^n - 2Q_i^n + Q_{i-1}^n}{\Delta x^2} \right) = 0 \tag{4}$$

further simplifying Equation (4) we obtain:

$$\frac{dQ_i^n}{dt} = Q_{i-1}^n \left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2} \right) - Q_i^n \left(\frac{2D}{\Delta x^2} \right) - Q_{i+1}^n \left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2} \right) \tag{5}$$

The next step is to apply the 4th order Runge Kutta method to obtain a numerical solution to Equation (1), that is:

$$Q^{n+1} = Q^n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{6}$$

with:

$$k_1 = \Delta t f(t_n, Q^n)$$

$$k_2 = \Delta t f\left(t_n + \frac{1}{2}\Delta t, Q^n + \frac{1}{2}k_1\right)$$

$$k_3 = \Delta t f\left(t_n + \frac{1}{2}\Delta t, Q^n + \frac{1}{2}k_2\right)$$

$$k_4 = \Delta t f(t_n + \Delta t, Q^n + k_3)$$

where, $f(t_n, Q_i^n)$ is a function defining an equation that can be expressed as follows:

$$f(t_n, Q_i^n) = Q_{i-1}^n\left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2}\right) - Q_i^n\left(\frac{2D}{\Delta x^2}\right) - Q_{i+1}^n\left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2}\right)$$

then, simplify the values k_1, k_2, k_3 and k_4 to obtain the simple form of Equation (6).

For k_1:

$$k_1 = \Delta t\left(Q_{i-1}^n\left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2}\right) - Q_i^n\left(\frac{2D}{\Delta x^2}\right) - Q_{i+1}^n\left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2}\right)\right)$$

for k_2:

$$k_2 = \Delta t\left(Q_{i-1}^n\left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2}\right) - \left(Q_i^n + \frac{1}{2}k_1\right)\left(\frac{2D}{\Delta x^2}\right) - Q_{i+1}^n\left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2}\right)\right)$$

for k_3:

$$k_3 = \Delta t\left(Q_{i-1}^n\left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2}\right) - \left(Q_i^n + \frac{1}{2}k_2\right)\left(\frac{2D}{\Delta x^2}\right) - Q_{i+1}^n\left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2}\right)\right)$$

for k_4:

$$k_4 = \Delta t\left(Q_{i-1}^n\left(\frac{\lambda}{2\Delta x} + \frac{D}{\Delta x^2}\right) - (Q_i^n + k_3)\left(\frac{2D}{\Delta x^2}\right) - Q_{i+1}^n\left(\frac{\lambda}{2\Delta x} - \frac{D}{\Delta x^2}\right)\right)$$

after obtaining the simple form of k_1, k_2, k_3 and k_4, then define the form of this equation into Equation (6), so that we obtain:

$$
\begin{aligned}
Q_i^{n+1} = {} & Q_i^n - (Q_{i+1}^n - Q_{i-1}^n)\left(\frac{\lambda\Delta t}{2\Delta x}\right) + (Q_{i+1}^n + Q_{i-1}^n)\left(\frac{D\Delta t}{\Delta x^2}\right) - Q_i^n\left(\frac{2D\Delta t}{\Delta x^2}\right) \\
& + (Q_{i+1}^n - Q_{i-1}^n)\left(\frac{\lambda\Delta t}{2\Delta x}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) - (Q_{i+1}^n + Q_{i-1}^n)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) \\
& + Q_i^n\left(\frac{2D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) - (Q_{i+1}^n - Q_{i-1}^n)\left(\frac{2\lambda\Delta t}{6\Delta x}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) \\
& + (Q_{i+1}^n + Q_{i-1}^n)\left(\frac{2D\Delta t}{3\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) - Q_i^n\left(\frac{2D\Delta t}{3\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{2D\Delta t}{\Delta x^2}\right) \\
& + (Q_{i+1}^n - Q_{i-1}^n)\left(\frac{\lambda\Delta t}{6\Delta x}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) \\
& - (Q_{i+1}^n + Q_{i-1}^n)\left(\frac{D\Delta t}{3\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right) \\
& + Q_i^n\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{D\Delta t}{\Delta x^2}\right)\left(\frac{2D\Delta t}{3\Delta x^2}\right)
\end{aligned} \tag{7}
$$

3.2 Stability Analysis

The stability test will be carried out using von Neumann analysis for a conservative form of diffusion wave flood tracking model with the MOL. Von Neumann stability can be found by substituting the value $Q_i^n = B^{(n)}e^{ikm\Delta x}$ into Equation (7). So we obtain:

$$|B| = \sqrt{1} = 1$$

Based on the criteria for the stability requirements of the finite difference method $|B| \leq 1$, it can be said that the completion of the diffusion wave flood tracking model using the MOL meets the stability requirements.

3.3 Program Simulation

The first simulation will be carried out using flow velocities of 2 m/s, 4 m/s and 6 m/s with a flow width of 50 m, flow depth of 4 m and pias of 20.

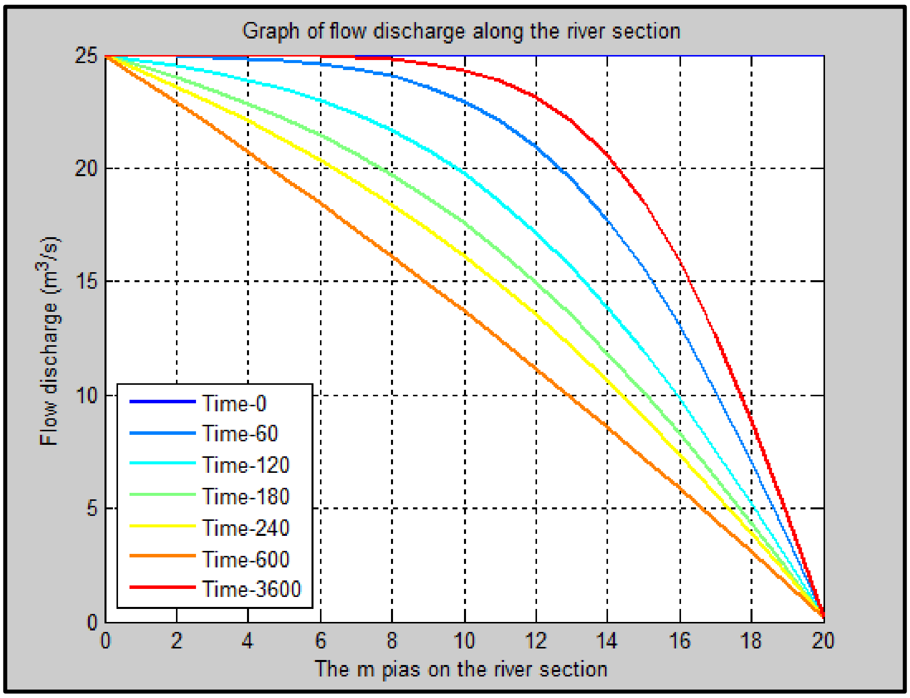

Figure 1. Graph of Flow Discharge at Velocity $2m/s$, Width $50m$ and Depth $4m$

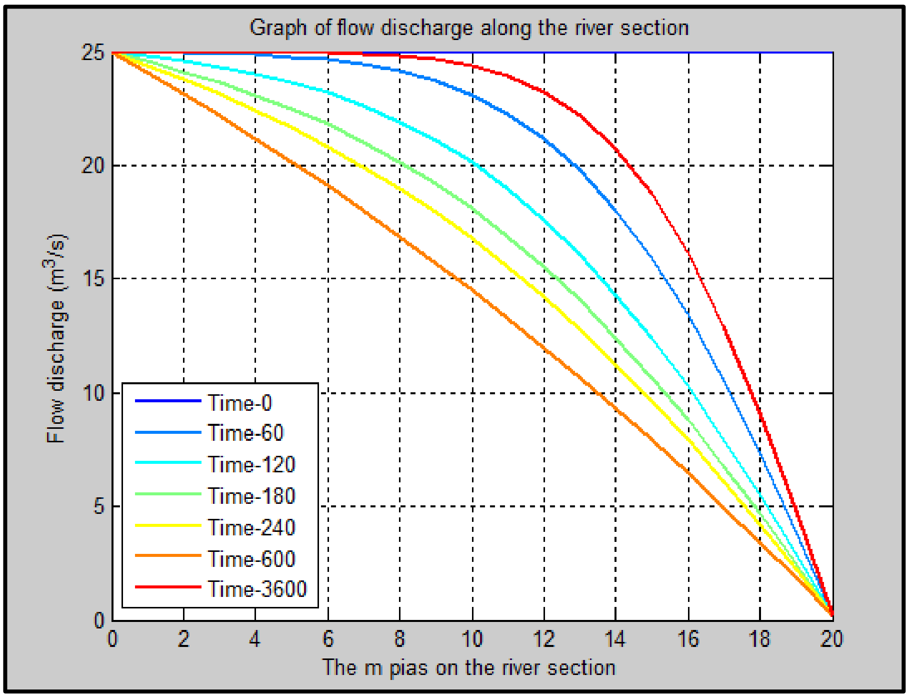

Figure 2. Graph of Flow Discharge at Velocity $4m/s$, Width $50m$ and Depth $4m$

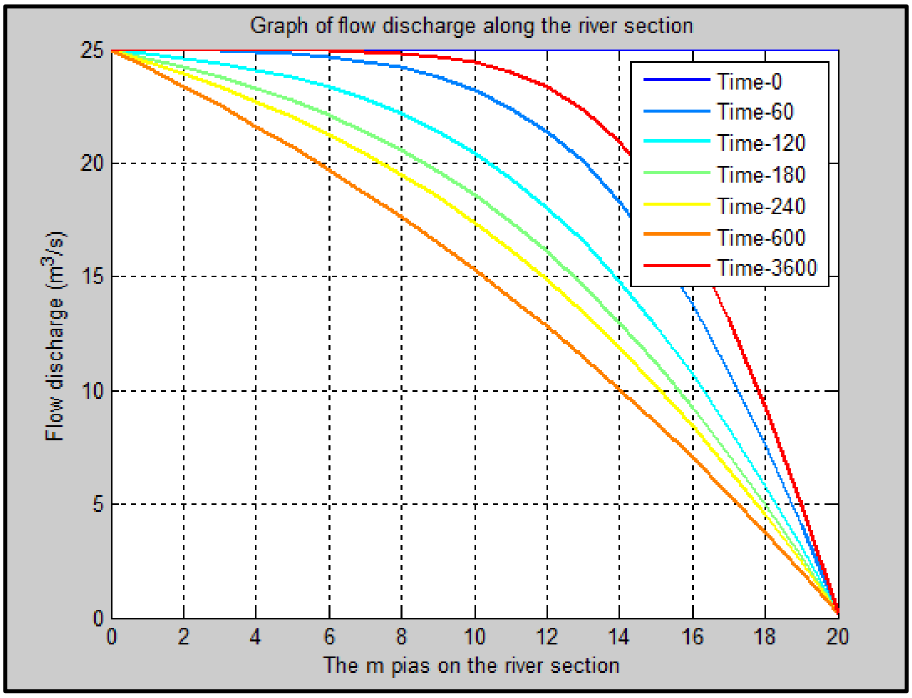

Figure 3. Graph of Flow Discharge at Velocity $6m/s$, Width $50m$ and Depth $4m$

The graph of the effect of flow velocity on flow discharge can be seen in Figure 1, Figure 2 and Figure 3 which shows that the flow discharge decreases when the flow $x = 0\,m$ towards the flow direction $x = 15.000\,m$. So the effect of flow velocity on the flow rate shows that with different flow velocities it can be seen that if the flow velocity is greater, the resulting water flow rate will be greater. This is in accordance with the physical law $Q = Av$, which means that the flow rate is directly proportional to the flow velocity. This means that the greater the flow velocity (v), the greater the flow rate (Q) produced if the cross-sectional area (A) used remains constant.

The second simulation will be carried out using flow width of $100\,m$ with a flow velocities of $2\,m/s$, $4\,m/s$ and $6\,m/s$, flow depth of $4\,m$ and pias of 20.

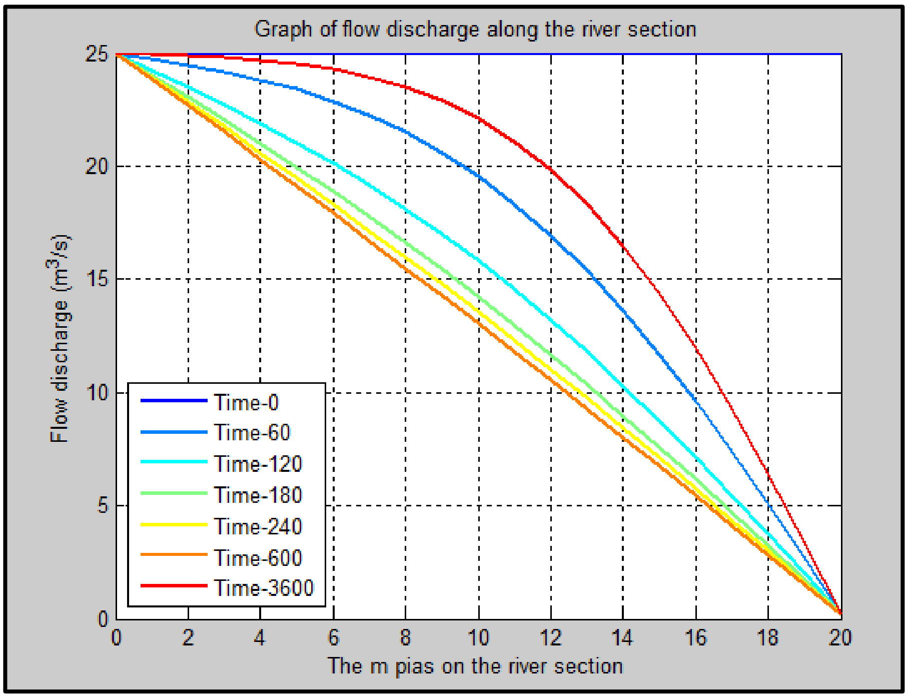

Figure 4. Graph of Flow Discharge at Width $100m$, Velocity $2m/s$ and Depth $4m$

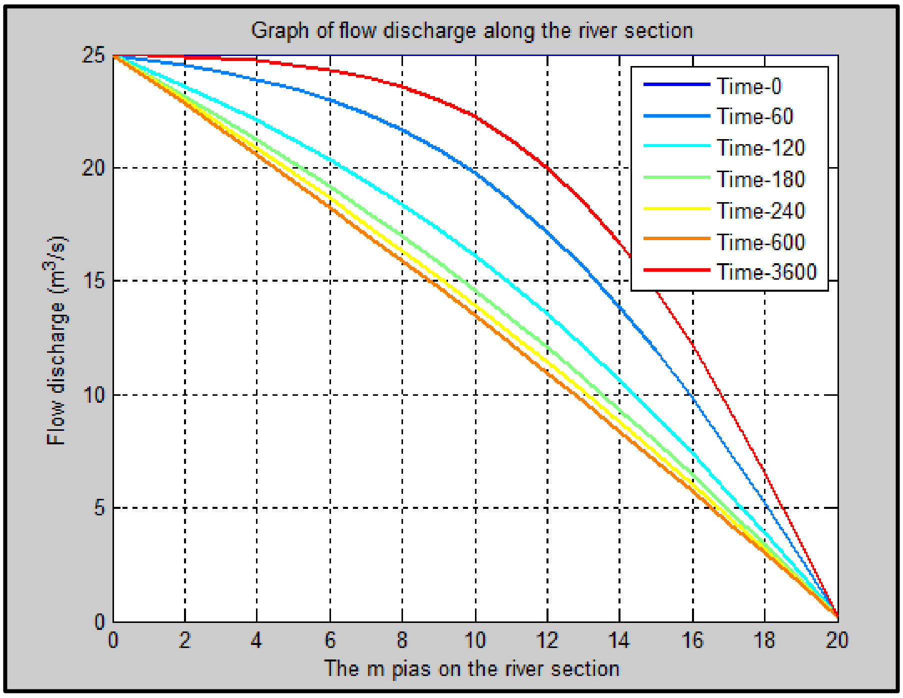

Figure 5. Graph of Flow Discharge at Width $100m$, Velocity $4m/s$ and Depth $4m$

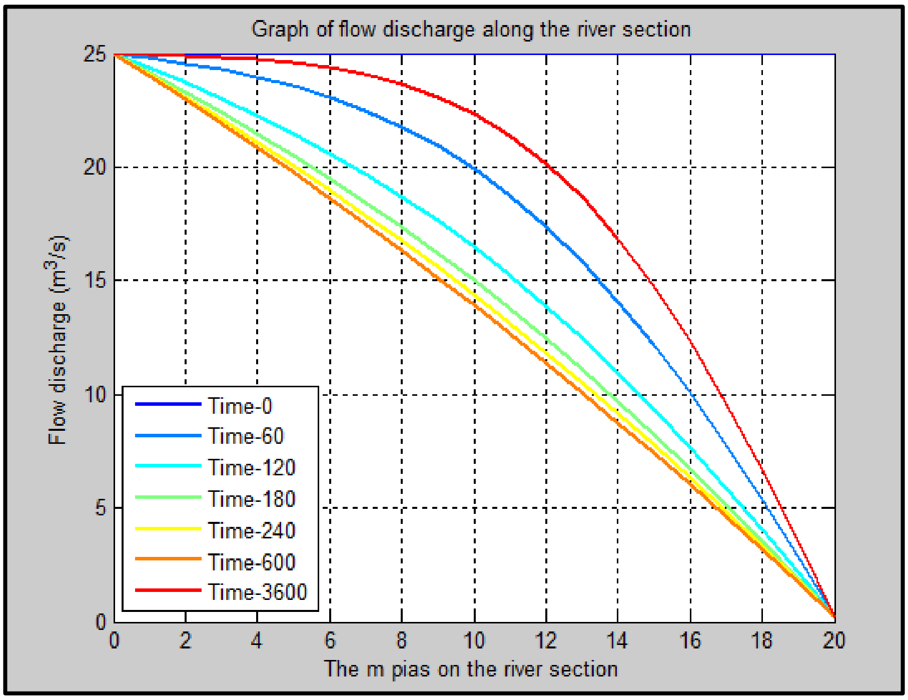

Figure 6. Graph of Flow Discharge at Width $100m$, Velocity $6m/s$ and Depth $4m$

The graph of the effect of flow width on flow discharge can be seen in Figure 4, Figure 5 and Figure 6 which shows that the flow discharge decreases when the flow $x = 0\ m$ towards the flow $x = 15.000\ m$. So the effect of different stream widths on stream discharge can be seen that the greater the stream width, the smaller the stream flow produced along the river section. Conversely, the smaller the width of the stream, the greater the flow rate produced along the river section.

The third simulation will be carried out using flow depth of 4 m with a flow velocities of 2 m/s, 4 m/s and 6 m/s, flow width of 50 m and pias of 20.

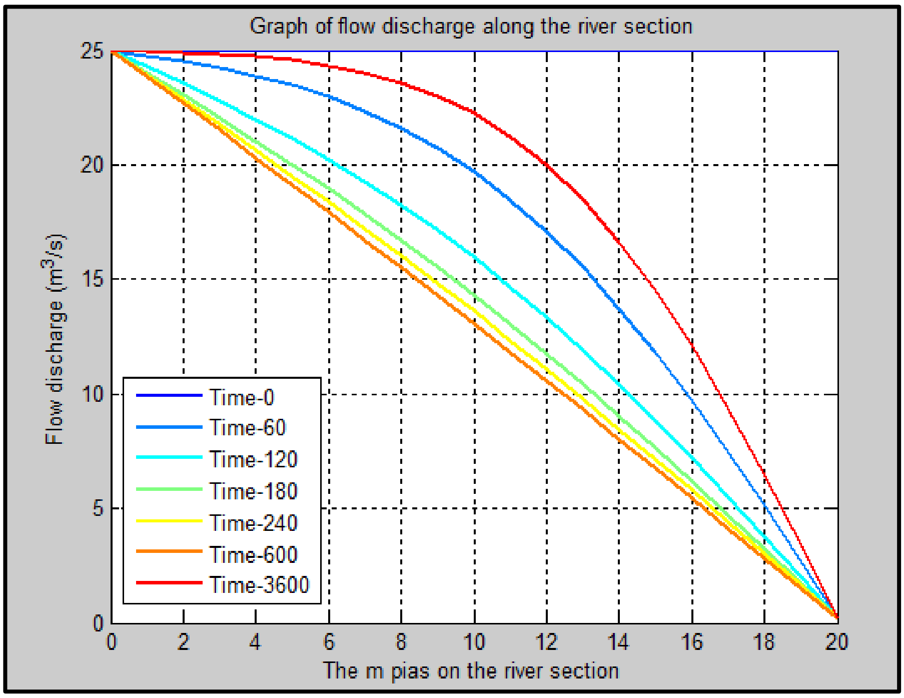

Figure 7. Graph of Flow Discharge at Depth $5m$, Velocity $2m/s$ and Width $50m$

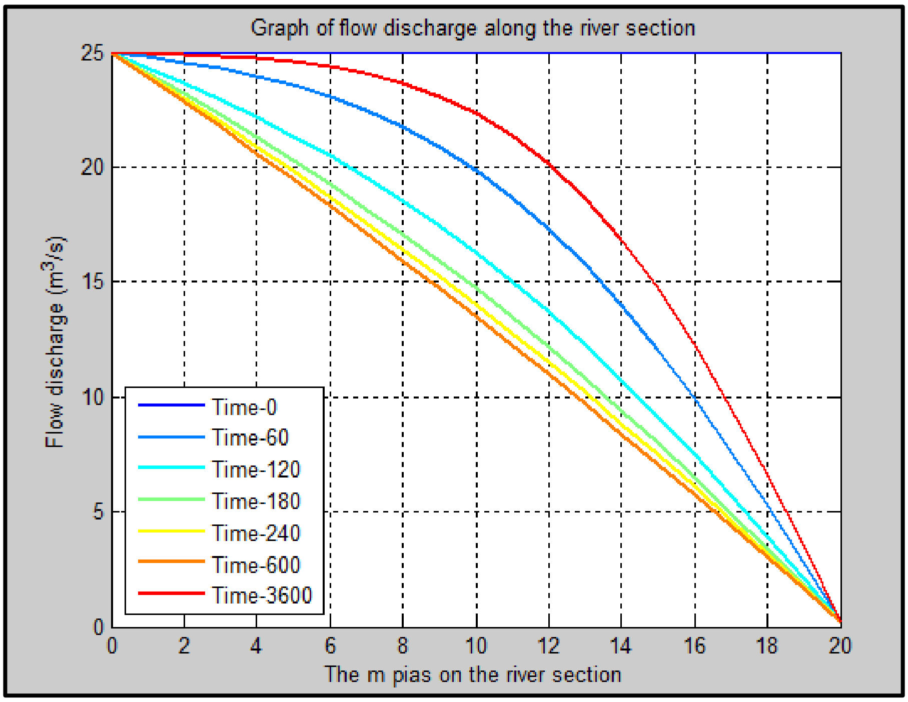

Figure 8. Graph of Flow Discharge at Depth $5m$, Velocity $4m/s$ and Width $50m$

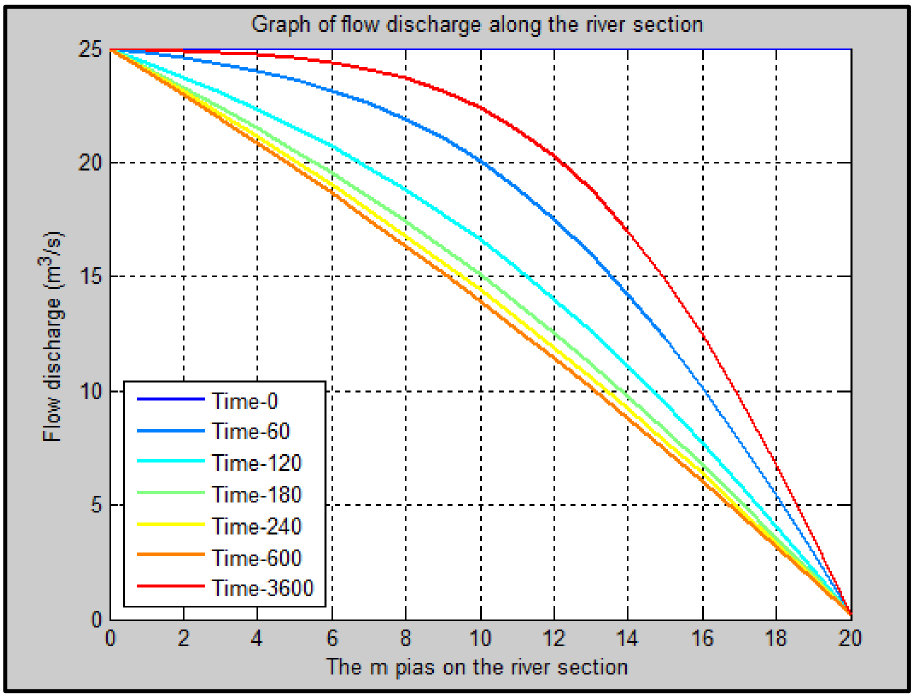

Figure 9. Graph of Flow Discharge at Depth $5m$, Velocity $6m/s$ and Width $50m$

The graph of the influence of flow width on flow discharge can be seen in Figure 7, Figure 8 and Figure 9 which shows that the resulting graph is decreasing over time. So, based on the influence of different flow depths on the flow discharge, it can be seen that the deeper the river flow, the narrower the flow rate produced along the river section. Conversely, the narrower the river flow, the deeper the resulting flow discharge.

4. CONCLUSIONS

The simulation results of the diffusion wave flood tracking model using the Method of Lines (MOL) can be concluded that the average flow velocity, flow width and flow depth influence the flow discharge of flood waves.

Different flow velocities, if the flow velocity increases, the water discharge downstream of the river will increase more rapidly. Likewise, if the flow speed decreases, the water discharge downstream of the river will take longer to increase.

The width and depth of the flow are different, it is found that the greater the width and depth of the flow, the resulting flow discharge will be smaller and closer to the downstream of the river. Likewise, if the width and depth of the flow is smaller, the resulting flow discharge will be greater and further away from the river downstream.

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