JSDS: JOURNAL OF STATISTICS AND DATA SCIENCE
VOLUME 1, No 1, March 2022

https://ejournal.unib.ac.id/index.php/jsds/index

Partitioned Design Matrix Method for Two Factors Multivariate Design

Renny Alvionita*, Sigit Nugroho, Mohammad Chozin
Mathematics Department, Bengkulu University, Indonesia

* Corresponding Author. E-mail: renny.alvionita@gmail.com

Article Info

Article History:
Received: December 10, 2021
Revised: December 17, 2021
Accepted: December 22, 2021
Available Online: March 1, 2022

Key Words:
Partitioned Design Matrix,
Sum of Product Matrices,
Degrees of Freedom.

Abstract

Factorial experiment often involves large data sets and the use of generalized inverse for the data analysis. It becomes less manageable as the data increased. The objective of this study is to evaluate the accuracy of partitioned design matrix method for two factors multivariate design. The design matrix is partitioned into several sub-matrices based on their source of variation. The partitioned design matrix method in two factors multivariate is much simpler than usual sigma summation method in calculating the sum of product matrix and the degrees of freedom. This method could also be used in explaining the derivation of the statistics for testing the hypothesis of the equality of the means which corresponds to the source of variation.

1. INTRODUCTION

Analysis of variance can be performed on one or more response variables. For one response variable, it is called univariate analysis of variance or often known as ANOVA. Whereas for more than one response variable, it can be done in two ways, namely, (1) performing ANOVA on each response variable separately, called multiple ANOVA, and (2) through multivariate analysis of variance or called MANOVA [13].

Multiple ANOVA is performed if there is no correlation between the response variables. However, multiple ANOVA has several drawbacks, for example, (i) it cannot see the effect of several treatment variables on the response variable in the form of constructs [3], and (ii) it will increase the possibility of making type I errors, namely rejecting H_0 when H_0 is true. These shortcomings will have consequences that cause inaccuracies in interpreting the results and drawing conclusions [2]. In such conditions, MANOVA can be used as an alternative because with MANOVA, the response variable which is a construct can be evaluated in its entirety.

The calculation of the sum of squares for both univariate and multivariate can be done by using elementary algebra notation as well as matrix algebra. However, for the multivariate, the use of elementary algebra will be very complex and error-prone. This causes the use of matrix algebra notation as a solution [7]. The calculation process using matrix algebra notation can be done simultaneously through matrix operations.

A popular method for calculating the sum of squares by matrix operations is the General Linear Model (GLM). By using this method, the level of the treatment variable is converted into a dummy variable, and the total number of squares is calculated using a projection matrix based on the generalized inverse [1]. However, when the data is large, the use of generalized inverse will provide a longer calculation process compared to using the usual inverse. To overcome this, the partitioned design matrix method can be used [6].

The partitioned design matrix method is a method used to determine the number of squares using the GLM with the design matrix partitioned according to the source of diversity. So far, the effectiveness of this method still needs to be clarified, especially for multivariate two-factor experiments. The purpose of this study was to evaluate the accuracy of the partitioned matrix design method in a two-factor multivariate experiment.

2. RESULTS AND DISCUSSION

In a two-factor multivariate experiment, there were n random observations where $n \geq 1$ was given to two treatment factors, namely A and B. Each treatment factor had the a level and the b level which formed a design with a combination of ab treatments. Then a test is carried out to test whether there is a difference in treatment on the p variable for the two treatment factors [12].

2.1 The Usual Sigma Summation Method

Two-way balanced multivariate two-factor experimental linear model with a constant effect for the dependent variable p is [8]

$$\mathbf{y}_{ijk} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\beta}_j + \boldsymbol{\gamma}_{ij} + \boldsymbol{\varepsilon}_{ijk} = \boldsymbol{\mu}_{ij} + \boldsymbol{\varepsilon}_{ijk} \tag{1}$$

where $\mathbf{y}_{ijk}$ for $i = 1,2,\dots,a$; $j = 1,2,\dots,b$; $k = 1,2,\dots,n$ is the observation vector for the k-th repetition given to factor B at the j-th level and factor A at the i-level, $\boldsymbol{\mu}$ is the general average vector, $\boldsymbol{\mu}_{ij}$ is the average vector for factor A at the i-level and factor B at the j-level, $\boldsymbol{\alpha}_i$ is the influence vector for factor A at the i-level, $\boldsymbol{\beta}_j$ is the influence vector for factor B at the j-th level, $\boldsymbol{\gamma}_{ij}$ is vector of the effect of the interaction AB, and $\boldsymbol{\varepsilon}_{ijk}$ is the component vector of the error in the k-th repetition observation given to factor B at the j-th level and factor A to the i-level.

The assumptions needed in this experiment are:

1. $\sum_{i=1}^{a} \boldsymbol{\alpha}_i = \mathbf{0}$, is the sum of all effects of factor A equal to zero.
2. $\sum_{j=1}^{b} \boldsymbol{\beta}_j = \mathbf{0}$, is the sum of all effects of factor B equal to zero.
3. $\sum_{i=1}^{a} \boldsymbol{\gamma}_{ij} = \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij} = \mathbf{0}$, is the sum of all interactions AB equal to zero
4. $\boldsymbol{\varepsilon}_{ijk} \sim N_P(\mathbf{0}, \boldsymbol{\Sigma})$, is an independent observation error with a normal distribution with a mean of zero and a certain variance.

Furthermore, the algebraic formula used to calculate the sum of squares matrix in a two-factor multivariate experiment is as follows:

Correction Factor : $\mathbf{F_K} = \dfrac{\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\boldsymbol{y}_{ijk}\right)\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\boldsymbol{y}_{ijk}\right)^t}{abn} = \dfrac{1}{abn}\boldsymbol{y}_{\dots}\boldsymbol{y}_{\dots}^t$

Sum of Squares of Factor A : $\mathbf{H_A} = \dfrac{1}{bn}\sum_{i=1}^{a}\boldsymbol{y}_{i..}\boldsymbol{y}_{i..}^{\boldsymbol{t}} - \mathbf{F_K}$

Sum of Squares of Factor B : $\mathbf{H_B} = \dfrac{1}{an}\sum_{j=1}^{b}\boldsymbol{y}_{.j.}\boldsymbol{y}_{.j.}^{\boldsymbol{t}} - \mathbf{F_K}$

Sum of Squares of Interaction AB : $\mathbf{H_{AB}} = \dfrac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\boldsymbol{y}_{ij.}\boldsymbol{y}_{ij.}^{\boldsymbol{t}} - \mathbf{F_K} - \mathbf{H_A} - \mathbf{H_B}$

Sum of total squares : $\mathbf{T} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\boldsymbol{y}_{ijk}\boldsymbol{y}_{ijk}^{\boldsymbol{t}} - \mathbf{F_K}$

Sum of Squares error : $\mathbf{E} = \mathbf{T} - \mathbf{H_A} - \mathbf{H_B} - \mathbf{H_{AB}}$

2.2 The Partitioned Design Matrix Method

The general linear model (GLM) of equation (1) is as follows:

$$\mathbf{Y}_{abn\times p} = \mathbf{X}_{abn\times(1+a+b+ab)}\boldsymbol{\beta}_{(1+a+b+ab)\times p} + \boldsymbol{\varepsilon}_{abn\times p} \tag{2}$$

where $\mathbf{Y}_{abn\times p}$ is an observation matrix of size $abn \times p$, $\mathbf{X}_{abn\times(1+a+ab)}$ is a design matrix of size $abn \times (1 + a + b + ab)$ which is partitioned into several sub-matrixes, namely $[\mathbf{X}_\mu|\mathbf{X}_\alpha|\mathbf{X}_\beta|\mathbf{X}_{\alpha\beta}]$. Each sub-matrix can be described as follows:

$$\mathbf{X}_\mu = \mathbf{1}_{a\times 1} \otimes \mathbf{1}_{b\times 1} \otimes \mathbf{1}_{n\times 1} \qquad \mathbf{X}_\beta = \mathbf{1}_{a\times 1} \otimes \mathbf{I}_{b\times b} \otimes \mathbf{1}_{n\times 1}$$
$$\mathbf{X}_\alpha = \mathbf{I}_{a\times a} \otimes \mathbf{1}_{b\times 1} \otimes \mathbf{1}_{n\times 1} \qquad \mathbf{X}_{\alpha\beta} = \mathbf{I}_{a\times a} \otimes \mathbf{I}_{b\times b} \otimes \mathbf{1}_{n\times 1}$$

and $\boldsymbol{\beta}_{(1+a+b+ab)\times p}$ is the model parameter matrix with size $(1 + a + b + ab) \times p$, $\boldsymbol{\beta}^{\mathrm{t}}_{p\times(1+a+b+ab)} = (\boldsymbol{\mu}, \boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_a, \boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_b, \boldsymbol{\gamma}_{11}, \dots, \boldsymbol{\gamma}_{1b}, \dots, \boldsymbol{\gamma}_{ab})$ and $\boldsymbol{\varepsilon}_{abn\times p}$ are experimental error matrices of size $abn \times p$. Before discussing the use of matrix notation on the sum of the squares in each component, we first calculate the projection matrix with the general form $\mathbf{M} = \mathbf{X}(\mathbf{X}^{\mathrm{t}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{t}}$ as follows:

$$\mathbf{M}_\mu = \frac{1}{abn}\mathbf{J}_{a\times a} \otimes \mathbf{J}_{b\times b} \otimes \mathbf{J}_{n\times n} \qquad \mathbf{M}_\beta = \frac{1}{an}\mathbf{J}_{a\times a} \otimes \mathbf{I}_{b\times b} \otimes \mathbf{J}_{n\times n}$$
$$\mathbf{M}_\alpha = \frac{1}{bn}\mathbf{I}_{a\times a} \otimes \mathbf{J}_{b\times b} \otimes \mathbf{J}_{n\times n} \qquad \mathbf{M}_{\alpha\beta} = \frac{1}{n}\mathbf{I}_{a\times a} \otimes \mathbf{I}_{b\times b} \otimes \mathbf{J}_{n\times n}$$

The projection matrix multiplication table can be seen in Table 1.

Table 1. Projection matrix multiplication

	$\mathbf{M}_\mu$	$\mathbf{M}_\alpha$	$\mathbf{M}_\beta$	$\mathbf{M}_{\alpha\beta}$
$\mathbf{M}_\mu$	$\mathbf{M}_\mu$	$\mathbf{M}_\mu$	$\mathbf{M}_\mu$	$\mathbf{M}_\mu$
$\mathbf{M}_\alpha$	$\mathbf{M}_\mu$	$\mathbf{M}_\alpha$	$\mathbf{M}_\mu$	$\mathbf{M}_\alpha$
$\mathbf{M}_\beta$	$\mathbf{M}_\mu$	$\mathbf{M}_\mu$	$\mathbf{M}_\beta$	$\mathbf{M}_\beta$
$\mathbf{M}_{\alpha\beta}$	$\mathbf{M}_\mu$	$\mathbf{M}_\alpha$	$\mathbf{M}_\beta$	$\mathbf{M}_{\alpha\beta}$

By using matrix notation, the matrix of the sum of squares of each component of the multivariate two-factor experiment variance can be written as:

$$\mathbf{F_K} = \mathbf{Y^t}\mathbf{M}_\mu\mathbf{Y}$$
$$\mathbf{H}_A = \mathbf{Y^t}(\mathbf{M}_\alpha - \mathbf{M}_\mu)\mathbf{Y}$$
$$\mathbf{H}_B = \mathbf{Y^t}(\mathbf{M}_\beta - \mathbf{M}_\mu)\mathbf{Y}$$
$$\mathbf{H}_{AB} = \mathbf{Y^t}(\mathbf{M}_{\alpha\beta} - \mathbf{M}_\alpha - \mathbf{M}_\beta + \mathbf{M}_\mu)\mathbf{Y}$$
$$\mathbf{T} = \mathbf{Y}^t(\mathbf{I} - \mathbf{M}_\mu)\mathbf{Y}$$
$$\mathbf{E} = \mathbf{Y^t}(\mathbf{I} - \mathbf{M}_{\alpha\beta})\mathbf{Y}$$

Theorem 1[11]:

Let $\mathbf{Y^t}$ be a matrix of size $m \times n$ whose columns are independent, with the i-th column having the distribution $N_m(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is positive definite. Suppose that $\mathbf{A}$ and $\mathbf{B}$ are symmetric matrices of size $n \times n$ while $\mathbf{C}$ is a matrix of size $k \times n$. Let $\mathbf{M^t} = (\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_n)$, $\boldsymbol{\Phi} = \frac{1}{2}\mathbf{M^t A M}$, and r = rank($\mathbf{A}$), then

(a) $\mathbf{Y^t A Y} \sim W_m(\boldsymbol{\Sigma}, r, \boldsymbol{\Phi})$, if $\mathbf{A}$ is idempotent,
(b) $\mathbf{Y^t A Y}$ and $\mathbf{Y^t B Y}$ are mutually independent if $\mathbf{AB} = \mathbf{O}$,
(c) $\mathbf{Y^t A Y}$ and $\mathbf{CY}$ are mutually independent if $\mathbf{CA} = \mathbf{O}$.

By using the properties of the projection matrix and the matrix multiplication table, it can be shown that $(\mathbf{M}_\alpha - \mathbf{M}_\mu)$, $(\mathbf{M}_\beta - \mathbf{M}_\mu)$, $(\mathbf{M}_{\alpha\beta} - \mathbf{M}_\alpha)$, $(\mathbf{M}_{\alpha\beta} - \mathbf{M}_\alpha - \mathbf{M}_\beta + \mathbf{M}_\mu)$, $(\mathbf{I} - \mathbf{M}_{\alpha\beta})$, and $(\mathbf{I} - \mathbf{M}_\mu)$ are symmetric and idempotent matrices. Thus, the rank of each matrix is the same as its respective trace [10]. Therefore, we get:

Degree of freedom of factor *A* : $\mathrm{tr}(\mathbf{M}_\alpha - \mathbf{M}_\mu) = a - 1$
Degree of freedom of factor *B* : $\mathrm{tr}(\mathbf{M}_\beta - \mathbf{M}_\mu) = b - 1$
Degree of freedom of interaction *AB* : $\mathrm{tr}(\mathbf{M}_{\alpha\beta} - \mathbf{M}_\alpha - \mathbf{M}_\beta + \mathbf{M}_\mu) = (a-1)(b-1)$

Degree of freedom of error : $\text{tr}(\mathbf{I}-\mathbf{M}_{\alpha\beta}) = ab(n-1)$
Total of degree of freedom : $\text{tr}(\mathbf{I}-\mathbf{M}_{\mu}) = abn-1$

Based on Theorem 1(a), without lost of generality, if $\mathbf{Y} \sim N_p(\mathbf{0},\mathbf{\Sigma})$, then $\mathbf{H}_{\boldsymbol{A}} \sim W_p(\mathbf{\Sigma}, a-1)$, $\mathbf{H}_{\boldsymbol{B}} \sim W_p(\mathbf{\Sigma}, a-1)$, $\mathbf{H}_{\boldsymbol{AB}} \sim W_p(\mathbf{\Sigma},(a-1)(b-1))$, and $\mathbf{E} \sim W_p(\mathbf{\Sigma}, ab(n-1))$.

When $\mathbf{A} \sim W_p(\mathbf{\Sigma}, m)$ and $\mathbf{B} \sim W_p(\mathbf{\Sigma}, n)$ also $\mathbf{A}$ and $\mathbf{B}$ are independent, then $\Lambda = \frac{|\mathbf{A}|}{|\mathbf{A}+\mathbf{B}|}$ has a Wilks' lambda distribution with parameters p, m and n [4]. The eigenvalue of $\mathbf{A}^{-1}\mathbf{B}$ is $\lambda_1 > \cdots > \lambda_p$, where $\mathbf{A} \sim W_p(\mathbf{\Sigma}, m)$ is independent of $\mathbf{B} \sim W_p(\mathbf{\Sigma}, n, \mathbf{\Phi})$ and $s = \min(p, n)$, then three test statistics can be used, namely:

$$V = \text{tr}[(\mathbf{E}+\mathbf{H})^{-1}\mathbf{H}] = \sum_{i=1}^{s} \frac{\lambda_i}{1+\lambda_i} \quad ; \quad U = \text{tr}(\mathbf{E}^{-1}\mathbf{H}) = \sum_{i=1}^{s} \lambda_i$$

and the largest root [5, 8]. The biggest square root statistic is

$$\theta = \frac{\lambda_1}{1+\lambda_1}$$

where λ_1 is the largest eigenvalue of $\mathbf{A}^{-1}\mathbf{B}$ [4].

To verify that the sum of squares of errors is independent of each matrix of the sum of the squares of the principal effects and their interactions, Theorem 1(b) can be applied and it can be checked with the help of the projection matrix multiplication table, that is $(\mathbf{I}-\mathbf{M}_{\alpha\beta})(\mathbf{M}_{\alpha}-\mathbf{M}_{\mu}) = \mathbf{0}$, $(\mathbf{I}-\mathbf{M}_{\alpha\beta})(\mathbf{M}_{\beta}-\mathbf{M}_{\mu}) = \mathbf{0}$, and $(\mathbf{I}-\mathbf{M}_{\alpha\beta})(\mathbf{M}_{\alpha\beta}-\mathbf{M}_{\alpha}-\mathbf{M}_{\beta}+\mathbf{M}_{\mu}) = \mathbf{0}$.

It can be concluded that

1. To test the main effect of factor *A*, reject the null hypothesis if the value $\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{H}_A|}$ is less than the value of $\Lambda_{p,ab(n-1),a-1}$ or when $V = \text{tr}\left[(\mathbf{E}+\mathbf{H}_A)^{-1}\mathbf{H}_A\right] = \sum_{i=1}^{s}\frac{\lambda_i}{1+\lambda_i}$, $U = \text{tr}\left(\mathbf{E}^{-1}\mathbf{H}_A\right) = \sum_{i=1}^{s}\lambda_i$, and $\theta = \frac{\lambda_1}{1+\lambda_1}$ has a relatively large value,
2. To test the main effect of factor B, reject the null hypothesis if the value $\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{H}_B|}$ is smaller than the value of $\Lambda_{p,ab(n-1),b-1}$ or when $V = \text{tr}\left[(\mathbf{E}+\mathbf{H}_B)^{-1}\mathbf{H}_B\right] = \sum_{i=1}^{s}\frac{\lambda_i}{1+\lambda_i}$, $U = \text{tr}\left(\mathbf{E}^{-1}\mathbf{H}_B\right) = \sum_{i=1}^{s}\lambda_i$, and $\theta = \frac{\lambda_1}{1+\lambda_1}$ has a relatively large value,
3. To test the main effect of the *AB* factor, reject the null hypothesis if the value $\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{H}_{AB}|}$ is smaller than the value of $\Lambda_{p,ab(n-1),(a-1)(b-1)}$ or when $V = \text{tr}\left[(\mathbf{E}+\mathbf{H}_{AB})^{-1}\mathbf{H}_{AB}\right] = \sum_{i=1}^{s}\frac{\lambda_i}{1+\lambda_i}$, $U = \text{tr}\left(\mathbf{E}^{-1}\mathbf{H}_{AB}\right) = \sum_{i=1}^{s}\lambda_i$, and $\theta = \frac{\lambda_1}{1+\lambda_1}$ has a relatively large value.

3. NUMERICAL EXAMPLE

The example used in the multivariate two-factor experimental design is taken from the book "Methods of Multivariate Analysis" by Rencher [9]. Table 2 shows the data on chickpeas, which are the results of four variables, namely y_1 = early harvest, y_2 = initial specific leaf area (SLA), y_3 = total yield, y_4 = average SLA. The factors used are planting date (*A*) and variety (*B*).

Before completing the solution using the usual sigma addition method or the partitioned design matrix, the correlation test on the response variables was first tested using the Bartlett test. The following is a correlation matrix of response variables and Bartlett's test obtained using the R program:

$$R = \begin{bmatrix} 1.000 & -0.848 & 0.870 & -0.501 \\ -0.848 & 1.000 & -0.924 & 0.603 \\ 0.870 & -0.924 & 1.000 & -0.698 \\ -0.501 & 0.603 & -0.698 & 1.000 \end{bmatrix}$$

$$\chi^2_{hit} = -\left[N - 1 - \frac{2p+5}{6}\right]\ln|R| = 236.4434$$

Because $\chi^2_{hit} = 236.4434 > \chi^2_{0.05,6} = 12.592$, it can be concluded that at the 5% level, there is not enough evidence to accept H_0. This conclusion means that the response variables correlate so that the analysis process can be continued.

Table 2. Data of chickpeas

S	V		y_1	y_2	y_3	y_4	S	V		y_1	y_2	y_3	y_4
1	1	1	59.3	4.5	38.4	295	3	1	1	68.1	3.4	42.2	280
		2	60.3	4.5	38.6	302			2	68.0	2.9	42.4	284
		3	60.9	5.3	37.2	318			3	68.5	3.3	41.5	286
		4	60.6	5.8	38.1	345			4	68.6	3.1	41.9	284
		5	60.4	6.0	38.8	325			5	68.6	3.3	42.1	268
1	2	1	59.3	6.7	37.9	275	3	2	1	64.0	3.6	40.9	233
		2	59.4	4.8	36.6	290			2	63.4	3.9	41.4	248
		3	60.0	5.1	38.7	295			3	63.5	3.7	41.6	244
		4	58.9	5.8	37.5	296			4	63.4	3.7	41.4	266
		5	59.5	4.8	37.0	330			5	63.5	4.1	41.1	244
1	3	1	59.4	5.1	38.7	299	3	3	1	68.0	3.7	42.3	293
		2	60.2	5.3	37.0	315			2	68.7	3.5	41.6	284
		3	60.7	6.4	37.4	304			3	68.7	3.8	40.7	277
		4	60.5	7.1	37.0	302			4	68.4	3.5	42.0	299
		5	60.1	7.8	36.9	308			5	68.6	3.4	42.4	285
2	1	1	63.7	5.4	39.5	271	4	1	1	69.8	1.4	48.4	265
		2	64.1	5.4	39.2	284			2	69.5	1.3	47.8	247
		3	63.4	5.4	39.0	281			3	69.5	1.3	46.9	231
		4	63.2	5.3	39.0	291			4	69.9	1.3	47.5	268
		5	63.2	5.0	39.0	270			5	70.3	1.1	47.1	247
2	2	1	60.6	6.8	38.1	248	4	2	1	66.6	1.8	45.7	205
		2	61.0	6.5	38.6	264			2	66.5	1.7	46.8	239
		3	60.7	6.8	38.8	257			3	67.1	1.7	46.3	230
		4	60.6	7.1	38.6	260			4	65.8	1.8	46.3	235
		5	60.3	6.0	38.5	261			5	65.6	1.9	46.1	220
2	3	1	63.8	5.7	40.5	282	4	3	1	70.1	1.7	48.1	253
		2	63.2	6.1	40.2	284			2	72.3	0.7	47.8	249
		3	63.3	6.0	40.0	291			3	69.7	1.5	46.7	226
		4	63.2	5.9	40.0	299			4	69.9	1.3	47.1	248
		5	63.1	5.4	39.7	295			5	69.8	1.4	46.7	236

Factor *A* is the date of planting and factor *B* is variety. The results of data analysis on a multivariate two-factor experiment using the usual sigma addition method can be seen as follows:

Sum of squares of Factor A : $\boldsymbol{H}_A = \begin{bmatrix} 728.790 & -352.488 & 690.115 & -4563.785 \\ -352.488 & 195.865 & -370.454 & 2187.423 \\ 690.115 & -370.454 & 747.776 & -4741.505 \\ -4563.785 & 2187.423 & -4741.505 & 33469.383 \end{bmatrix}$

Sum of squares of Factor B : $\boldsymbol{H}_B = \begin{bmatrix} 124.521 & -16.135 & 32.098 & 1008.583 \\ -16.135 & 4.866 & -4.756 & -137.958 \\ 32.098 & -4.756 & 8.402 & 261.548 \\ 1008.583 & -137.958 & 261.548 & 8188.233 \end{bmatrix}$

Sum of squares of interaction AB : $\boldsymbol{H}_{AB} = \begin{bmatrix} 30.295 & -6.027 & 2.956 & 130.710 \\ -6.027 & 4.912 & -2.904 & -38.988 \\ 2.956 & -2.904 & 5.867 & 59.665 \\ 130.710 & -38.988 & 59.665 & 1887.767 \end{bmatrix}$

Sum of total squares : $\mathbf{T} = \begin{bmatrix} 895.502 & -374.596 & 725.062 & -3378.872 \\ -374.596 & 217.807 & -379.716 & 2004.457 \\ 725.062 & -379.716 & 775.702 & -4379.272 \\ -3378.872 & 2004.457 & -4379.272 & 50790.983 \end{bmatrix}$

Sum of squares error : $\mathbf{E} = \begin{bmatrix} 11.896 & 0.054 & -0.108 & 45.620 \\ 0.054 & 12.164 & -1.602 & -6.020 \\ -0.108 & -1.602 & 13.656 & 41.020 \\ 45.62 & -6.020 & 41.020 & 7245.600 \end{bmatrix}$

Then the respective degrees of freedom (*db*) are:

$$db[A] = a - 1 = 4 - 1 = 3$$
$$db[B] = b - 1 = 3 - 1 = 2$$
$$db[AB] = (a-1)(b-1) = (4-1)(3-1) = (3)(2) = 6$$
$$db[error] = ab(n-1) = (3)(4)(5-1) = 48$$
$$db[total] = abn - 1 = (4)(3)(5) - 1 = 59$$

The results of hypothesis testing can be seen in Table 3:

Table 3. The results of the four test statistics and the F test approach

		Λ	θ	V	U
Factor A	Stat. Test	0.001	0.993	2.359	146.107
	F	121.36	1515.135	44.2	725.122
	F_{table}	1.824	2.589	1.820	1.95
Factor B	Stat. Test	0.066	0.920	1.104	11.670
	F	32.68	123.034	14.77	134.209
	F_{table}	2.579	2.589	2.036	2.20
Interaction AB	Stat. Test	0.135	0.729	1.334	3.501
	F	4.95	18.354	3.84	6.345
	F_{table}	1.577	2.33	1.577	1.58

To test the effect of factor A, it has been found that the results of the *F* test approach from the four test statistics have a value greater than the value of F_{table}, so there is not enough evidence to accept $\boldsymbol{H_{0A}}$. This means that at the 5% level of significance, factor A, namely the date of planting, has a significant effect on initial yield, initial specific leaf area (SLA), total yield, and average SLA. Furthermore, to test the effect of factor B and interaction AB can be done in the same way so that it is obtained that there is not enough evidence to accept $\boldsymbol{H_{0B}}$ and $\boldsymbol{H_{0AB}}$. Therefore, it can be concluded that at the 5% level of significance, factor B, namely the variety has a significant influence on the initial yield, initial (SLA), total yield, and average SLA and there is also an interaction between planting date and variety.

While the results of data analysis using a partitioned design matrix obtained with the help of the R program are as follows:

```
Source  DF      SSP.1    SSP.2     SSP.3     SSP.4 Lambda Pillai  Lawley   Roy
               728.79 -352.488   690.115 -4563.785
   A     3   -352.488  195.865  -370.454  2187.423  0.001  2.359 146.107 0.993
              690.115 -370.454   747.776 -4741.505
```

		-4563.785	2187.423	-4741.505	33469.383				
		124.521	-16.135	32.098	1008.583				
B	2	-16.135	4.866	-4.756	-137.958	0.066	1.104	11.67	0.92
		32.098	-4.756	8.402	261.548				
		1008.583	-137.958	261.548	8188.233				
		30.295	-6.027	2.956	130.71				
AB	6	-6.027	4.912	-2.904	-38.988	0.135	1.334	3.501	0.729
		2.956	-2.904	5.867	59.665				
		130.71	-38.988	59.665	1887.767				
		11.896	0.054	-0.108	45.62				
Galat	48	0.054	12.164	-1.602	-6.02				
		-0.108	-1.602	13.656	41.02				
		45.62	-6.02	41.02	7245.6				
		895.502	-374.596	725.062	-3378.872				
Total	59	-374.596	217.807	-379.716	2004.457				
		725.062	-379.716	775.702	-4379.272				
		-3378.872	2004.457	-4379.272	50790.983				

From the results of testing the influence of factor A, factor B, and interaction AB above, it can be seen that the four test statistics of each factor obtained using a partitioned design matrix have the same results as calculations using the usual sigma addition method. Therefore, that the results of each *F*-test approach will also be the same. It can be concluded that the partitioned design matrix method has the same effectiveness and accuracy as the usual sigma addition method.

4. CONCLUSION

The calculation of the sum of squares and degrees of freedom matrix in a multivariate two-factor experiment using the partitioned design matrix method is much straightforward than the usual sigma addition method. The use of a partitioned design matrix is operationally simple because in this method the design matrix used is partitioned into several sub-matrixes and has a similar shape that can be represented as Kronecker multiplication. In addition, there is no need to use generalized inverse matrices. One can use ordinary inverse matrices instead.

REFERENCES

[1] Christensen, R. 2011. Plane Answer to Complex Questions: The Theory of Linear Models, New York, Springer, 2011.

[2] Haase, R. F. and M, V. Ellis., “Multivariate Analysis of Variance”, Journal of Counselling Psychology. Vol. 34, No. 4, 1987.

[3] Huberty, C. J. and J. D. Morris. 1989. “Multivariate Analysis Versus Multiple Univariate Analyses”, Psychological Bulletin. Vol. 105, No. 2, 1989.

[4] Mardia, K. V., J. T. Kent, and J. M. Bibby., Multivariate Analysis, London, Academic Press, 1979.

[5] Muirhead, R. J., Aspects of Multivariate Statistical Theory, John Wiley & Sons. New York, 1982.

[6] Nugroho, S., “Analisis Keragaman Percobaan Tersarang dengan Menggunakan Matriks Rancangan Terpartisi”, Prosiding Semirata 2015 bidang MIPA BKS-PTN Barat, 2015.

[7] Nugroho, S., “Penggunaan Matriks Rancangan Terpartisi dalam Percobaan Tiga Faktor”, PRISMA 1 (2018).

[8] Rencher, A. C., . Multivariate Statistical Inference and Applications. Canada, John Wiley & Sons, 1998.

[9] Rencher, A. C., Methods of Multivariate Analysis, Second Edition, Canada, John Wiley & Sons, 2001.

[10] Rencher, A. C. and G. B. Schaalje, Linear Models in Statistics, Second Edition, New Jersey, John Wiley & Sons, 2008.

[11] Schott, J. R., Matrix Analysis for Statistics. Canada, John Wiley & Sons, 1997.

[12] Timm, N. H., Applied Multivariate Analysis. New York, Springer, 2002.

[13] Warne, R. T., “A Primer on Multivariate Analysis of Variance (MANOVA) for Behavior Scientists”, Practical Assessment Research & Evaluation. Vol. 19, No. 17, 2014