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Control Chart of T^2-Hotelling on Quality Control Activities of Crude Palm Oil (CPO) at PT Cipta Graha Garwita, Seluma Regency, Bengkulu Province

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Abstract

PT Cipta Graha Garwita (CGG) is a palm oil producer focused on product quality, especially crude palm oil (CPO) for both food and non-food applications. Despite CGG's good reputation, variability in quality characteristics such as Free Fatty Acid (FFA) and moisture can affect the final quality of CPO. This study aims to apply a statistical quality control system to monitor and improve the consistency of CPO quality using T^2-Hotelling control charts. Statistical quality control methods ensure that products meet standards by reducing variability. One such tool is the T^2-Hotelling control chart, effective for monitoring multivariate variables using mean vectors and variance-covariance matrices. This study involves steps from data collection, testing multivariate normality assumptions, calculating T^2-Hotelling control charts, to determining control limits. Testing for multivariate normality assumptions showed the data met normal distribution criteria. The first and second stage T^2-Hotelling control charts identified several out-of-control observations. These out-of-control observations were excluded, and further analysis showed that after their removal, all data were within statistical control limits. This study recommends further analysis to determine the causes of out-of-control observations using Ishikawa diagrams and process capability evaluation to ensure consistent product quality.

1. INTRODUCTION

PT Cipta Graha Garwita (CGG) is a palm oil producer committed to producing high-quality products according to industry standards. One of their main products is crude palm oil (CPO), used in various food and non-food industrial applications. Despite CGG's good reputation for product quality, changes in critical variables such as Free Fatty Acid (FFA) and moisture content can affect the final quality of their CPO. CGG has never statistically controlled these variables. Therefore, the researchers aim to implement an advanced quality control system using a statistical approach.

Statistical quality control is a set of methods used to ensure that products or services meet requirements by reducing variability. It is one of the greatest technological developments of the 20th century, based on strong principles, easy to use, has a significant impact, and can be applied to any process. Statistical quality control can be performed using tools commonly known as the magnificent seven or Seven Tools of Quality: histograms, check sheets, Pareto diagrams, cause-and-effect diagrams (fishbone/Ishikawa diagrams), defect concentration diagrams, scatter plots, and control charts. Control charts involving a single quality characteristic are called univariate control charts, while those involving two or more quality characteristics are called multivariate control charts. The T^2-Hotelling control chart is a multivariate control chart used to monitor quality control activities [4].

The purpose of this reseach is to implement T^2-Hotelling control charts combined with a statistical quality control system to monitor and enhance the consistency of CPO quality. The T^2-Hotelling control chart is used to monitor process mean shifts using mean vectors and variance-covariance matrices. It can monitor real-time variations in quality variables such as FFA and moisture during the CPO production process. Thus, researchers can provide recommendations to the company in detecting deviations from set quality targets and take corrective actions to ensure product quality consistency. The next step will be to create an Ishikawa diagram to identify problems causing out-

of-control CPO products and finally analyze process capability to determine the production process's capability level [4].

2. METHOD

This study will present the T²-Hotelling control chart, a multivariate control chart. It monitors process mean shifts using mean vectors and variance-covariance matrices. There are two types of T²-Hotelling control charts based on the number of observations: the T²-Hotelling control chart for subgroup observations and the T²-Hotelling control chart for individual observations. The individual T²-Hotelling control chart refers to control charts with subgroups of size one, where each observation is treated individually.

The steps in using the T²-Hotelling control chart include:

1. **Data Collection:** The data used in this study is laboratory test data of CPO characteristics at PT CGG, Seluma, Indonesia. The variables to be studied are Free Fatty Acid (FFA) and Moisture Content.
2. **Testing for Multivariate Normality Assumption:** This is done by comparing squared distance values with the chi-square distribution quantile values with degrees of freedom p. The hypotheses are H_0 the data is multivariate normally distributed, and H_1 the data is not multivariate normally distributed. The test statistic is:

$$r_q = \frac{\sum_{i=1}^{m}\left(d_{(i)}^2 - \bar{d}^2\right)(q_i - \bar{q})}{\sqrt{\sum_{i=1}^{m}\left(d_{(i)}^2 - \bar{d}^2\right)^2}\sqrt{\sum_{i=1}^{m}(q_i - \bar{q})^2}} \quad (1)$$

   Where, $d_{(i)}^2$: is the squared distance value, $\bar{d}^2$: is the mean squared distance, q_i : is the quantile value, and $\bar{q}$: is the mean quantile value. The rejection criterion is to reject H_0 if $r_q > \chi^2_{\alpha(m-1)}$ with m as the sample size and α as the significance level (0.05) [3].
3. **Calculating the Mean and Covariance Matrix of the Data**.
4. **Calculating the T²-Hotelling Control Chart for Each Observation**.

$$T^2 = (\boldsymbol{x} - \bar{\boldsymbol{x}})'\boldsymbol{S}^{-1}(\boldsymbol{x} - \bar{\boldsymbol{x}}) \quad (2)$$

   Where x : is the vector of each quality characteristic value for the observation, $\bar{x}$: is the mean vector of each quality characteristic, and S^{-1}: is the inverse of the variance-covariance matrix [4].
5. **Determining Control Limits**.

$$UCL = \frac{p(m-1)(n-1)(m-1)^2}{mn - m - p + 1} F_{\alpha,p,mn-m-p+1} \quad (3)$$
$$LCL = 0$$

   Where, $\beta_{\alpha,\frac{p}{2},\frac{(m-p-1)}{2}}$ is the $1-\alpha$ quantile of the beta distribution with parameters $\frac{p}{2}$ and $\frac{m-p-1}{2}$, UCL is the Upper Control Limit, and LCL is the Lower Control Limit [4].
6. **Plotting the T²-Hotelling Control Chart:** Plot the T² values for each observation on the y-axis and the observation numbers on the x-axis, with a horizontal line representing the Upper Control Limit (UCL).

3. RESULTS AND DISCUSSION

3.1 Testing for Multivariate Normality Assumption

The multivariate normality assumption was tested by comparing squared distance values with chi-square distribution quantile values with degrees of freedom p. Based on calculations using the equation (1), $r_q = 0.45$ was obtained, which will be compared with the chi-square distribution quantile value $\chi^2_{\alpha(m-2)} = 113.14$. Since $r_q < \chi^2_{\alpha(m-2)}$ H_0 is not rejected, indicating the data meets the multivariate normality assumption.

3.2 T²-Hotelling Control Chart

Using the R program, the T²-Hotelling control chart was created by installing the required packages and inputting the data. The first stage T²-Hotelling control chart and Ellipse Chart to detect out-of-control observations are shown in Figures 1 and 2.

```
install.packages('IAcsSPCR')
library(IAcsSPCR)
install.packages('qcc')
library(qcc)
install.packages('QuantPsyc')
library(QuantPsyc)
```

```
da <- read.csv('C:/.../pH5datacgg.csv', header = TRUE)
```

```
X1<-matrix(da$FFA,nrow=92,byrow=T)
X2<-matrix(da$Moisture,nrow=92,byrow=T)
XQ <- list(X1 = X1, X2 = X2)
s <- mqcc(XQ, type = "T2",add.stats=T,title="T2 chart")
summary(s)
ellipseChart(s, show.id = T)
```

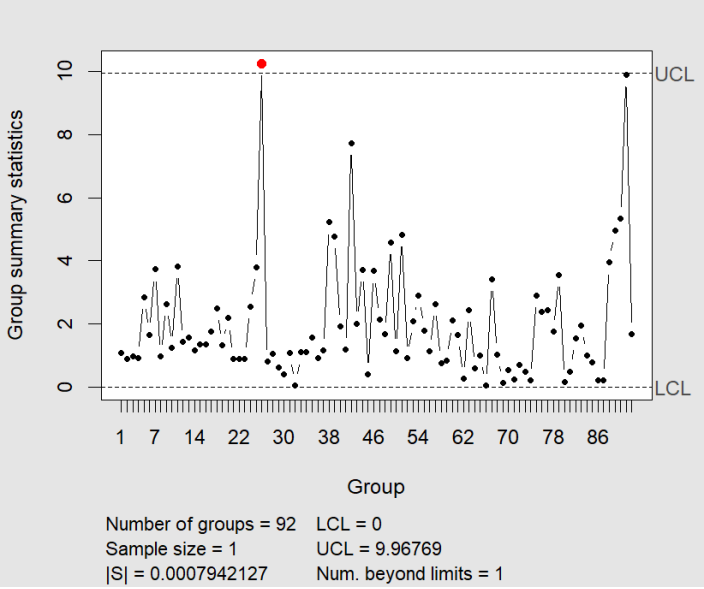

Figure 1. First Stage T^2-Hotelling Control Chart

Figure 1 indicates several out-of-control observations. Figure 2 shows which specific observations are out of control.

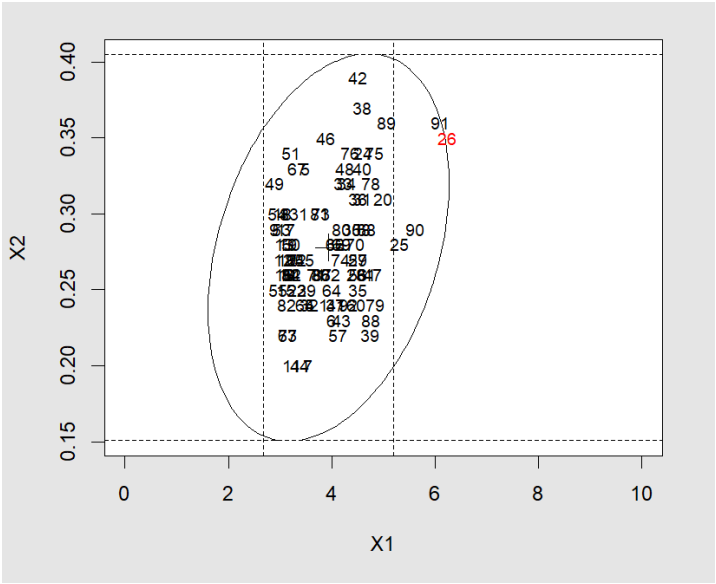

Figure 2. Ellipse Chart Stage 1

It is clear from Figure 2 that observation 26 out of 92 observations is out of control. Thus, the control process will be redone excluding observation 26.

```
# Remove OOC points
da_rmv26 <- da[-26, ]
X1 <- matrix(da_rmv$FFA,nrow=91,byrow=T)
X2 <- matrix(da_rmv$Moisture,nrow=91,byrow=T)
XQ2 = list(X1 = X1, X2 = X2)
s2 = mqcc(XQ2, type = "T2",add.stats=T,
          title="T2 chart without observation 26")
ellipseChart(s2, show.id = T)
```

Figure 3 showing T^2 Hotelling chart without observation26.

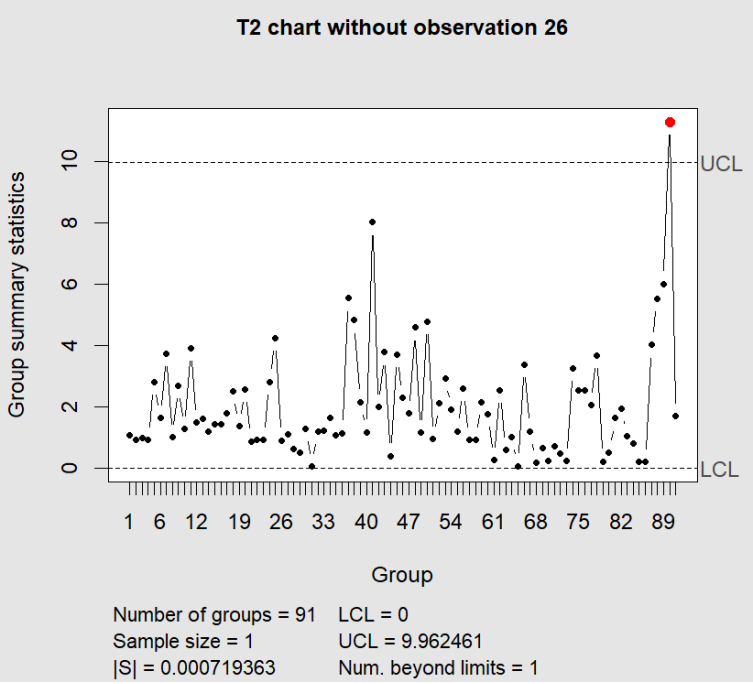

Figure 3. Second Stage T²-Hotelling Control Chart

Figure 3 indicates some observations are still out of control, requiring identification of out-of-control observations using the ellipse chart again. Figure 4 show observation which out of control in second stage.

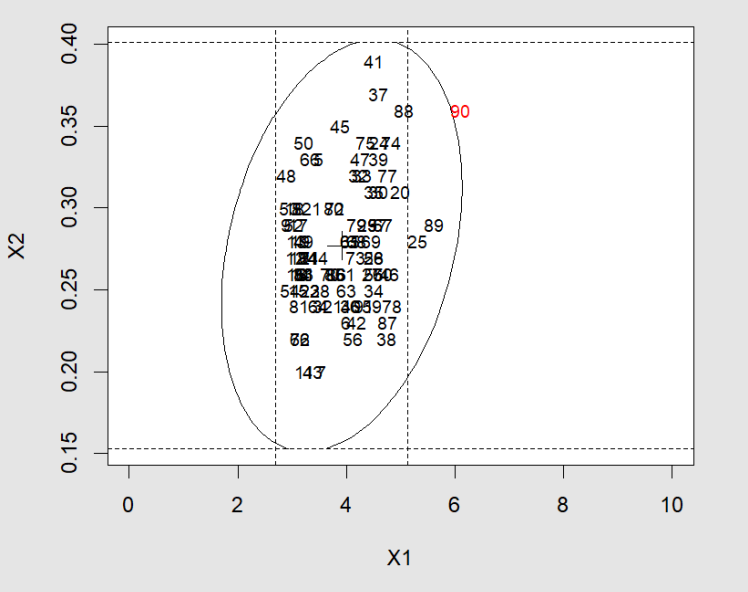

Figure 4. Ellipse Chart Stage 2

From Figure 4, observation 90 is identified as out of control. The T²-Hotelling control chart without observation 90 is redrawn.

```
da_rmv90 <- da_rmv26[-90, ]
X1 <- matrix(da_rmv90$FFA,nrow=90,byrow=T)
X2 <- matrix(da_rmv90$Moisture,nrow=90,byrow=T)
XQ3 = list(X1 = X1, X2 = X2)
s3 = mqcc(XQ3, type = "T2",add.stats=T,
          title="T2 chart without Observation 26 and 90")
```

Figure 5 show T^2 Hotteling control chart without observation 90th.

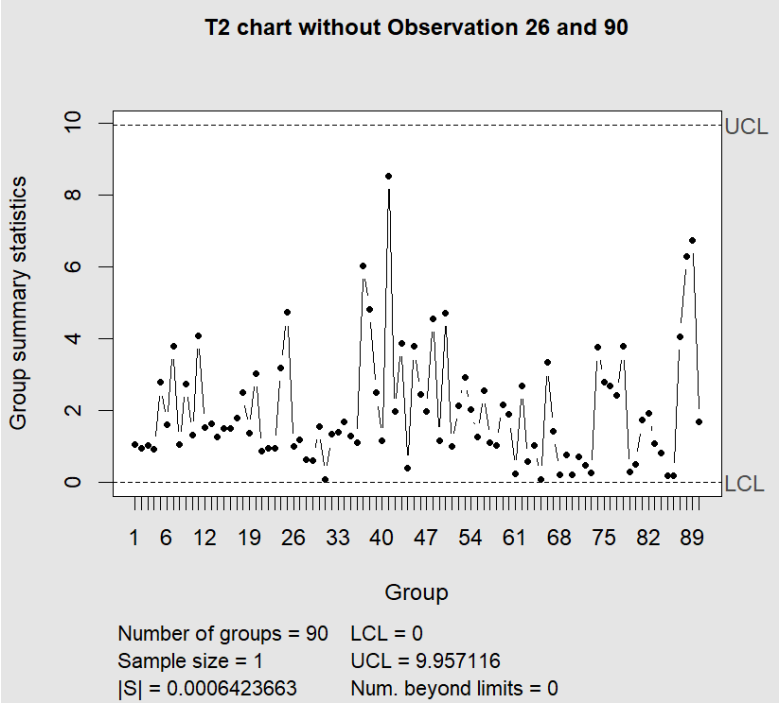

Figure 5. Third Stage T²-Hotelling Control Chart

Figure 5 shows all observations are within statistical control limits. Observation 26 and observation 90 will be analyzed in future research to determine the causes of these out-of-control observations.

4. CONCLUSION

The multivariate normality assumption test shows that the data meets the criteria for a multivariate normal distribution, allowing the creation of a T²-Hotelling multivariate control chart. The first and second stage T²-Hotelling control charts show some out-of-control observations, which are then excluded from the analysis, resulting in a control chart without any out-of-control observations. Future research should analyze the causes of these out-of-control observations using the Ishikawa diagram and calculate the process capability.

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