Cognitive Improvement in Thermodynamic Functions With Calculus Chain Rule

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ABSTRACT

This research was motivated by the low ability of students to apply mathematical solution methods to thermodynamic functions. Thermodynamic functions are functions that consist of pressure, temperature, volume and enthalpy functions where changes in one function can occur due to changes in other functions. The aim of this research was to show the implementation of chain rules in thermodynamic function problems by relying on a process technique to improve students' cognitive awareness of thermodynamic functions. The place of this research was at the TD Pardede Institute of Science and Technology. The process of understanding chain rules is very impactful for understanding thermodynamic state functions. Differential decomposition of functions within functions is also described in the chain rule method. Thermodynamic functions which consist of pressure, temperature, volume and enthalpy functions can depend on each other so that the chain rule analysis method can be clearly used. With this method students will also be given the opportunity to connect one concept with other thermodynamic concepts. Apart from that, this method can be used anywhere. This shows a cognitive increase in terms of cognitive elements, cognitive structure and cognitive function.

Keywords: Cognitive Improvement,Thermodynamic Functions, Chain Rule.

INTRODUCTION

Education is a conscious effort carried out by guiding, teaching and training students which can bring about changes in students. Learning is an important part that cannot be separated from the educational process. Learning is a process of changing behavior and thought patterns experienced by individuals. Cognitive enhancement learning theory emphasizes that what is most important in the learning process is the implementation of how the process occurs rather than the results achieved. Cognitive learning theory itself prioritizes the learning process rather than the results achieved. This process guarantee is what brings changes in thinking attitudes and results. Cognitive theory includes conscious mental activities such as thinking, knowing, understanding, and mental conception activities such as: attitudes, beliefs, and expectations, which then become determining factors in behavior. The theory of cognitive enhancement itself consists of three elements, namely the first, the cognitive element, which is directly related to the mind and consciousness (mental). Cognitive theory believes that a person's behavior is caused by a stimulus, namely a physical object that influences a person in many ways. The second is the cognitive structure where cognitive theory recognizes that the activity (process) of knowing and understanding something (cognition) does not stand alone. Experience and stimulus are structures that have a big influence on cognitive improvement itself. And the third cognitive function includes, among others, understanding, language, motivation and perception. Gestalt theory views learning as a process of understanding (insight) which is different from behaviorism theory which views learning as a process of trial and error. The definition of insight is sudden observation and understanding of the relationships between parts in a problem situation. Someone is said to be successful in the learning process if they gain insight. With insight, someone will understand the problems they face and be able to solve them. Basically, every individual behavior is based on cognition, namely the act of recognizing and thinking about the situation in which the behavior occurs. For example, in a learning situation, direct involvement in learning will make an individual understand so they can overcome existing problems. The implementation of cognitive enhancement is aimed at increasing learning experience, meaningfulness, purpose and life skills.

In studying physics courses, students encounter many difficulties in understanding and analyzing the material. This difficulty is due to basic physics analysis involving basic calculus (Nguyen & Rebello, 2011). This causes low understanding, analysis and student learning outcomes. The problem that occurs in learning is how to realize or implement cognitive processes in order to achieve an adequate cognitive level. Downstream from the achievement of this cognitive process can be seen from student learning outcomes. For example, in the application physics course (applied physics) is the application of physics to analytical and engineering problems. Where problems at this level require analytical methods such as mathematical analysis. Analytical mathematics such as calculus and algebra are needed to solve applied physics problems. Applied physics questions related to analytical mathematics relate to very small changes in physical variables as well as periodic questions. In a study, the results were that in learning applied physics, students often face solving problems related to applied mathematics (Simangunsong, S, & Trisna, I. (2021). In fact, there is a close connection between physics and mathematics so that mathematics requires the ability to understand physics concepts in mathematical models (Kereh, Liliasari, et al. 2014). For example, the pendulum solution is actually very easy to solve with the Taylor series. Preliminary studies reveal that many students' reasoning patterns are based on certain elements and actions carried out on the underlying mathematical elements. structure of physical problems. One of the materials that is difficult but is the basis for various fields of analytical science is thermodynamics. Thermodynamics has a fundamental role in everyday life and learning. As a basic science that needs to be understood well, teaching is needed Effective thermodynamics. Provision for good teaching can start with find out the difficulties faced by students regarding concepts Thermodynamics (Meltzer, 2004). The essence of thermodynamics is the study of changing energy into motion mechanics and business. To be able to understand the essence of thermodynamics requires understanding complete regarding macroscopic and microscopic quantities Based on the background above, the author took a stance to conduct research in examining the relationship between chain rules and cognitive improvement in thermodynamic functions.

RESEARCH METHOD

This research is qualitative research. This research is based on the emergence of several analytical physics learning problems whose solutions are close to mathematical methods. The data sources in this research were obtained through several journals and books regarding the application of the Chain Rule method in analytical physics problems. Chain Rule used in this research is one dimensional with a simple differential format. The applications of the Chain Rule described in this research are in Thermodynamic function as a function of condition. The place of this research was carried out at the TD Pardede Institute of Science and Technology where the researcher worked as a lecturer. This research was conducted for ten months.

RESULTS AND DISCUSSIONS

Chain rule by generalization

Chain rule in thermodynamics is a mathematical concept used to calculate the rate of change of thermodynamic properties as a function of several independent variables. Research has shown that one of the main reasons for the difficulty in learning the chain rule is because the chain rule is connected to various mathematical concepts. The genetic decomposition of the chain rule performed by Clark et al. (1997) supports this statement. This study shows that understanding chain rules at least involves the conception of function, composition and decomposition of function, as well as differentiation rules (Clark et al., 1997). Maharaj (2013) also reported that detecting the embedded functions inherent in a problem situation is a key step and difficult phase in learning the chain rule.

Therefore, we aim to find ways to adopt a modeling perspective to facilitate students' questions about chain rules starting with real-world problem situations. Lesh and Doerr (2003) argue that mathematical modeling activities can encourage students' exploration of the meaning of mathematical concepts or procedures in relation to real-world problem situations. As we have noted, because students' difficulty in learning the chain rule is a difficult problem to solve, research needs to find alternative ways to facilitate students' questions about the chain rule. Since a key aspect of the chain rule is explaining the combined ratio between variables (Guicciardini, 2003), it is easier to understand the chain rule in relation to real-world problem situations (Lutzer, 2003; Uygur & Ozdas, 2010). The chain rule is considered a particularly appropriate mathematical concept to explore using a modeling perspective because “calculus serves as the basis for modeling and problem solving in applications” (Tall, Smith, & Piez, 2008, p. 208). In the following section, we analyze the literature to examine how mathematical modeling can be applied to facilitate students' questions about the chain rule. Empirical studies show that mathematical modeling can activate students' questions regarding rates of change, derivatives, and function concepts . Thus, we first need to find theoretical factors and mechanisms while generalizing mathematical models that will achieve the objectives of this research and draw implications for further research in adapting real contexts in mathematics teaching and learning. To be more specific, we examine the Peircean perspective on generalization and design a mathematical modeling task that reflects its discussion. Peirce pointed out that generalization is a dynamic process based on “observation and hypothesis generation in specific cases” and “verification and revision of existing hypotheses” and is not a linear or gradual process (Otte, 2006). The key factor in this dynamic generalization process is abduction (Otte, 2006). In the following sections, we review the Peircean perspective on abduction and generalization to find ways to facilitate generalization of models and modeling activities.

In this study, we aimed to facilitate students' questions about the chain rule by supporting the generalization of modeling activities. We specifically focus on the use of analogies and diagrammatic reasoning to encourage students' abductive constructions. As a result, we determined that students used abduction based on analogical and diagrammatic reasoning, as well as building and generalizing mathematical models to derive chain rules. Although we cannot demonstrate the entire mechanism of model generalization with abduction, we reveal sub-mechanisms of model generalization and the use of abduction supported by analogies, diagrammatic reasoning, and real context.

The use of analogies by students is in the form of direct application of existing mathematical rules to new contexts, so that the use of these analogies results in undercoded abduction constructions. In other words, students rarely change existing rules when applying them to new contexts. For example, students directly apply “the relationship between area and edge length” and “the rule of polynomial differentiation” to a specific problem situation (Episode 1). Although analogies are known to play a key role in knowledge construction, as emphasized by Lee and Sriraman (2011), students may not use analogies productively at the start of their investigations. Students' use of diagrammatic reasoning also takes the form of direct application of conventional rules to diagrams. Although students applied existing mathematical rules to new contexts in inadequate ways, the use of analogies and diagrammatic reasoning supported their investigation of the chain rule. To be more specific, the results of analogical reasoning and diagrams support the existence of general rules that explain models and modeling processes in certain real situations. It also contributes to the formation of initial hypotheses based on the problem situation. This means that students are able to apply existing rules to new contexts, and the results are partly in accordance with their modeling activities. Therefore, they can determine the existence of general mathematical regularities or rules similar to existing rules that describe a particular situation.

On the other hand, mathematical modeling activities serve as evidence that can be used to verify the validity and consistency of the use of analogical reasoning and diagrams based on conventional rules, which encourages students' use. Students' difficulties in learning calculus are reported, so the need to find alternative ways to facilitate students' investigation of the key ideas of calculus. In this study, we confirmed that mathematical modeling supported students' investigation of the chain rule and that facilitation of the use of abduction promoted generalization of the model. One of the main issues of debate in mathematics teaching and learning is the conflict of opinion regarding the role of real context. The results of this research show that there is a positive role in real contexts when conventional rules are used together with analogical and diagrammatic reasoning. Although we partially confirmed the synergistic relationship between real context and the use of conventional rules in the first two discussion points, this is still debatable. In addition, the adaptability of some content areas to real contexts may vary. Further research involving a variety of modeling tasks in different content areas is recommended to verify the feasibility of including mathematical modeling in mathematics teaching and learning.

Suppose we have a continuous function z where the value of the function depends on the variables x and y. So we can write the differential of z is

$$\Delta z = \frac{\partial z}{\partial x}\partial x + \frac{\partial z}{\partial y}\partial y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

Where the values of $\epsilon_1$and ϵ_2 go to zero then the values of Δx and Δy will also go to zero. Then the differential equation above can be written as:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

In this case, whether x and y are independent variables or not, the total change in the z function can be written like the dz equation above.The next question is whether the variables x and y can be in the form of a function whose value depends on other variables, say x and y depend on the time variable t. how do we write the change in the z function. We can write the function z in the notation $z \to z(x_{(t)}, y_{(t)})$. Then we can write the total derivative dz with respect to t as:

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Where ${\partial x}/{\partial t}$ and ${\partial y}/{\partial t}$ are changes in variables x and y with respect to t. This is because each variable x and y is also a function of t.

Based on this experience and our understanding, we can determine the derivative of the function if x and y not only consist of variable t but consist of variables s and t, or we can write the function z in the notation $z \to z(x_{(t,s)}, y_{(t,s)})$Then we can write the total change in dz as $dx = \frac{\partial x}{\partial s}ds + \frac{\partial x}{\partial t}dt$ dan

$$dz = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial s}ds + \frac{\partial x}{\partial t}dt\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial s}ds + \frac{\partial y}{\partial t}dt\right)$$
$$= \frac{\partial z}{\partial x}\frac{\partial x}{\partial s}ds + \frac{\partial z}{\partial x}\frac{\partial x}{\partial t}dt + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}ds + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}dt$$

$dy = \frac{\partial y}{\partial s}ds + \frac{\partial y}{\partial t}dt$

$$du = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s}ds + \frac{\partial u}{\partial x}\frac{\partial x}{\partial t}dt + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}ds + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}dt + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s}ds + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}dt$$

Based on the analysis above, for three variables we can derive the total derivative du with a function consisting of $u(x_{(t,s)}, y_{(t,s)}, z_{(t,s)})$ namely

Thus, if there is a function with n variables, we can determine the general form of the chain rule. For example, if we write a function f consisting of n variables $x_1, x_2, x_3, \dots, x_n$ and each variable also depends on its value on other variables so it can be written $x_{1(t_1,t_2,t_3,\dots,t_n)} x_{2(t_1,t_2,t_3,\dots,t_n)}$ and so on $x_{n(t_1,t_2,t_3,\dots,t_n)}$ Then the total derivative of the df function is

$$df = \left(\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_1}dt_1 + \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_2}dt_2 + \cdots\right). + \left(\frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_1}dt_1 + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_2}dt_2 + \cdots\right) + \left(\frac{\partial f}{\partial x_3}\frac{\partial x_3}{\partial t_1}dt_1 + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_2}dt_2 + \cdots\right) + \cdots$$

Changes in The Thermodynamic Function

Thermodynamics is a study of the state function of a system. The state of the system is indicated by pressure, temperature, volume, heat and enthalpy (P,T,V,E,H).The chain rule has been considered a notorious concept in calculus, and researchers have demonstrated students' difficulty in learning the chain rule (Clark et al., 1997; Gordon, 2005; Maharaj, 2013). For example, Gordon (2005) notes that the chain rule is difficult to explain so that most students do not really understand where the rule comes from (p. 195). He also argued that the chain rule is difficult to represent in symbols and awkward to express in words, so that most students do not remember or apply it correctly. It has also been reported that most students are not aware that they are using the chain rule (Clark et al., 1997).

Chain rule in thermodynamics is a mathematical concept used to calculate the rate of change of thermodynamic properties as a function of several independent variables. It is based on the principle of conservation of energy and is used to analyze and predict the behavior of thermodynamic systems. How to apply the chain rule in thermodynamics. In thermodynamics, the chain rule is applied to calculate the total change in a thermodynamic property which is a function of several independent variables. This is done by taking the partial derivatives of the property with respect to each independent variable and then multiplying them.

The chain rule in thermodynamics is important because it allows us to analyze and predict the behavior of complex thermodynamic systems. It helps us understand how a change in one variable affects the system as a whole and allows us to make accurate predictions about the behavior of the system. The chain rule can be applied to all thermodynamic systems as long as the system can be described using mathematical equations and has many independent variables.

This is a basic concept in thermodynamics and is used in various applications. One of the limitations of the chain rule in thermodynamics is
the assumption that the system is in a state of equilibrium. This means that the system does not change over time and all variables are constant. In real-world systems, this may not always be the case, and the chain rule may not be accurate in predicting its behavior.

As an example, prove the ideal gas equation with the pressure function $P = \frac{nRT}{V}$, volume $V = \frac{nRT}{P}$ and temperature $T = \frac{PV}{nR}$ with the chain rule $\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$. Based on the derivative of each part we get that $\frac{\partial P}{\partial V} = -\frac{nRT}{V^2}$, $\frac{\partial V}{\partial T} = \frac{nR}{P}$ and $\frac{\partial T}{\partial P} = \frac{V}{nR}$.So if we multiply according to the chain rule above we get:

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -\frac{nRT}{V^2}.\frac{nR}{P}.\frac{V}{nR} = -1$$

So we get the ideal gas equation $PV = nRT$.

From the analysis above, we can create a simple chain rule application using the table 1 below.

Table 1. Simple Chain Rule Application

NO	Function	Form a chain rule	condition
1	$P = P(T,V)$	$dP = \frac{\partial P}{\partial T} dT + \frac{\partial P}{\partial V} dV$	For a constant temperature process during the process, $dP = \frac{\partial P}{\partial V} dV$
2	$S = S[V(T,V),T]$	$dS = \frac{\partial P}{\partial T} dT + \frac{\partial S}{\partial V} dV$	
3	$V = V[P,T(S,P)]$	$dV = \frac{\partial V}{\partial T} dT + \frac{\partial V}{\partial P} dP$	
4	$E = E(T,V)$	$\left(\frac{dE}{dV}\right)_P = \left(\frac{\partial E}{\partial T}\right)_V \frac{dT}{dV} + \left(\frac{\partial E}{\partial V}\right)_T$	With $\left(\frac{\partial E}{\partial T}\right)_V = C_v$

CONCLUSION

Cognitive learning theory is a learning theory that prioritizes processes the learning is compared with the results achieved. The most important thing in cognitive theory is insight or understanding of the existing situation environment so that individuals are able to solve problems faced and also how individuals think. The process of understanding chain rules is very impactful for understanding thermodynamic state functions.

Differential decomposition of functions within functions is also described in the chain rule method. Thermodynamic functions consisting of pressure, temperature, volume and enthalpy functions can depend on each other so that the chain rule analysis method can be clearly used. Understanding that thermodynamic functions can change with each other can be described through chain rule differential equations so that students are increasingly able to understand the concept of thermodynamics and its changes. With this method, students will also be given the opportunity to connect one concept with other thermodynamic concepts. A part from that, this method can be used anywhere.

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