Linier Perturbation Method Analytic Shear Wave Propagation In A Dispersive Medium

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ABSTRACT

This research was based on showing how the relationship of the perturbation method to the wave function entering the dispersive medium area. Disperse medium is a medium where when the wave enters the area, it experiences changes in wave shape and wave energy. The purpose of this study is to show how the perturbation method can be reduced to a term that is still linear with the assumption that the next term is very small so that the wave term is still linear to the wave change. This research is qualitative research. The place of this research is at the TD Pardede Institute of Science and Technology. From the analysis in this study, it was found that the wave experienced a decreasing intensity towards the depth of the dispersive medium with the assumption that the attenuation coefficient (μ) was still continuous along the depth of the material passed by the wave. In the case of seismic waves when the wave moves through the medium, its intensity decreases with distance. The presence of disturbances in the layer passed by the wave causes the shift of the material to experience a disturbance characteristic where the wave amplitude is getting smaller. This is because the absorption of wave energy by the particles of the dispersive medium continues to experience attenuation.

Keywords: Perturbation Method, Shear Wave, Medium, Dispersive.

INTRODUCTION

Light as an electromagnetic wave is an entity present in the universe. One of the largest sources of light for the earth is sunlight. Sunlight is very important for the continuity of the photosynthesis process carried out by phytoplankton. Productive phytoplankton are only found in the upper layers of water, where the light intensity is sufficient for photosynthesis to take place (Hou, Z., Okamoto, R. J., & Bayly, P. V., 2020). Light also plays a role in visualizing the environment for aquatic organisms. In many cases today the need for optical properties of light is a very important need for the development of quality of life. The use of optical properties is useful in the world of engineering such as mining, construction and in the medical field. One of the optical properties that is useful in the above fields is the problem of scattering and attenuation (Mari, J.-L, 2019).

The nature of the dispersive material when illuminated shows the existence of an attenuation process in the material. The magnitude of this property can be calculated through the attenuation coefficient. The attenuation coefficient is a description of how much the incoming light is reduced or lost compared to the energy of the incoming light on the surface. This medium can be solid or liquid (Aleksandra Risteska & Vlado Gicev, 2018). The quantity of light that experiences attenuation is equivalent to the amount of light absorbed and scattered. The attenuation process also causes light penetration to only penetrate the water column to a certain depth. Therefore, knowledge of the attenuation coefficient can be used to determine the characteristics of a water column. The intensity of radiation entering a material is not the same but experiences energy loss (Lossy Energy). The intensity of radiation entering the material depends on how deep the light enters (Saydalimov, A. S, 2022).

Radiation intensity is the transfer of radiation radiation per unit area emitted by a radiation source (Aziman, M., Hazreek, Z. A. M., Azhar, A. T. S., & Haimi, D. S., 2016). Radiation radiation is the amount of radiation energy emitted in the form of electromagnetic radiation per unit time (sec) (Man, X., Luo, Z., Liu, J., & Xia, B., 2019). The quantity that states the comparative constant between the magnitude of the intensity of radiation absorbed and the thickness of a material or material is called the attenuation coefficient (μ) (Guo, F., Dong, et al.2023). The relationship between wave intensity and attenuation coefficient is that the intensity decreases with the depth of the wave radiation. This can be written as

$$I = I_0 e^{-\int \mu dx}$$

To analyze the propagating wave in the medium we use Gaussian waves because the Gaussian wave packet is localized so that we can still determine the state function of the quantity such as mass density, attenuation or wave energy in a certain inner layer (Zvietcovich, et. al, 2018). Suppose we assume the Gaussian wave function propagating in the x-axis is

$$\Psi(x,t_0) = Ae^{ikx-\frac{x^2}{2\sigma^2}}$$

where $A = \frac{1}{\pi^{1/4}\sqrt{\sigma}}$ with integral transformation

$$\Psi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(x,t_0).\,e^{-ikx}dx$$

So that we get

$$\Psi(k) = \frac{A}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}}dx$$

By using the relationship

$$\int_{-\infty}^{\infty} e^{\gamma+\beta x-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}}e^{\frac{\beta^2}{4\alpha}+\gamma}$$

So we get $\Psi(k) = \frac{\sqrt{\sigma}}{\pi^{1/4}}$

By entering the function $\Psi(k)$ into the function $\Psi(x,t)$ *we get*

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(k).\,e^{i\left(kx-\frac{\hbar k^2 t}{2m}\right)}dk$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{\sigma}}{\pi^{1/4}}\int_{-\infty}^{\infty} e^{-\left(\frac{i\hbar t}{2m}\right)k^2+(ix)k}dk$$

By using the integral relationship above:

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{\sigma}}{\pi^{1/4}}\sqrt{\frac{2\pi m}{i\hbar t}}\,e^{\frac{(ix)^2}{\left(\frac{i\hbar t}{2m}\right)}}$$

$$= \frac{\sqrt{\sigma}m}{\pi^{1/4}i\hbar t}e^{\frac{-mx^2}{2i\hbar t}}$$

Seismic waves are waves that propagate in the earth caused by structural deformation, pressure or tension due to the elastic properties of the earth's crust. These waves carry energy and then propagate in all directions throughout the earth and can be recorded by seismographs. Surface waves are complex waves with low frequencies and large amplitudes, which propagate due to the free surface effect, namely the difference between the elastic properties. The propagation of seismic waves below the surface will be influenced by the properties of the heterogeneous and anisotropic medium. The propagation of seismic waves below the surface will be influenced by the properties of the heterogeneous and anisotropic medium (Shen, W., Ritzwoller, et al, 2016).

Heterogeneous properties are defined as the physical properties of a medium that depend on position so that they do not depend on direction, this is different from anisotropy which depends on direction (Luo, G., et. al 2023). This heterogeneous property has variations in physical properties on a small scale (grain scale) while anisotropy has variations in physical properties on a large scale. This scale is determined by a standard of comparison, namely the seismic wavelength. Anisotropic velocity affects the position laterally and at depth and the focus of geological structures. The success of seismic imaging is the calculation and estimation of accurate anisotropy.

The attenuation coefficient is also useful in the recognition of solid layers (rocks). The need to determine the identification of the attenuation coefficient for solid layers (rocks), so that layers that have hard or soft soil types can be identified. If the value of the sediment or rock attenuation coefficient in an area is large, then the vulnerability of the sediment to earthquakes is high or the area is dangerous when an earthquake occurs, and conversely if the value of the sediment attenuation coefficient obtained is small, then the area is an area with a lower risk when an earthquake occurs. For the field of geology, Shear wave velocity and shear modulus are common to determine the interaction of dynamic soil properties (Leng, K., Chintanapakdee, C., & Hayashikawa, T., 2014).

RESEARCH METHOD

The method used in this study is a literature study through various references. This study begins with a fundamental analysis of the nature of seismic shear waves. By using infinite frequencies, the function of the Fourier transform is proposed by adopting the frequency of the shear wave. The general intensity is given as a description of the shear signal with a general form in elastic waves, namely functions in real and imaginary. By entering the concept of first-term disturbance, the wave propagation function is obtained. This research was conducted for a year at the TD Pardede Institute of Science and Technology.

RESULTS AND DISCUSSION

The interaction of waves in a solid medium that experiences dispersion can be written in the form of the Maxwell equation (Cornille, P., Atomic, F., & Commission, E. 2011).

The Maxwell equation itself is an electromagnetic wave equation that shows changes in magnetic fields and electric fields. The equation in the medium can be written as;

$$\nabla\, x\, B = \mu\epsilon E + \mu\epsilon \frac{\partial E}{\partial t}$$

$$\nabla\, x\, (\nabla\, x\, E) = -\frac{\partial(\nabla\, x\, B)}{\partial t}$$

$$\nabla\,.\,(\nabla\,.\,E) - \nabla^2 E = -\frac{\partial\left(\mu\epsilon E + \mu\epsilon \frac{\partial E}{\partial t}\right)}{\partial t}$$

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

So as a consequence of the equation above, the wave equation can be written

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t},$$

$$\nabla^2 B = \mu\epsilon \frac{\partial^2 B}{\partial t^2} + \mu\sigma \frac{\partial B}{\partial t}$$

So the solution to the above equation in the wave plane is

$$E_{(z,t)} = E_0 e^{i(kz-\omega t)},\ B_{(z,t)} = Be^{i(kz-\omega t)}$$

With the wave number $\tilde{k}$ in complex form

$$\tilde{k} = k + i\kappa$$

with

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2},$$

$$\kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}$$

κ is the imaginary part that shows the disturbance of energy absorption in the medium which is proportional to the distance or depth the wave propagates in the medium (Cáceres, M.O, 2021). The interaction of waves propagating in a medium can be shown in a Gaussian function in the axial direction, namely:

$$I_{(x)} = \left(1/_{\sigma}\right)e^{-\left(x^2/_{2\sigma^2}\right)}$$

$I_{(x)}$ is a wave propagation with a low attenuation level. The Gaussian integral transform of $I_{(x)}$ is

$$I_{(\omega)} = \sqrt{2\pi}e^{-\left(x^2\omega^2/_2\right)}$$

The wave speed propagating in the x direction can be written in the form of the Gaussian amplitude P.

$$v_z(x,t) = \frac{Pc^2}{2\sigma}e^{\left(-(x-ct)^2/_{2\sigma^2}\right)}$$

Here for the Gaussian value $P = 1$ then

$$v_z(x,\omega) = \frac{\omega}{2k} e^{-\frac{1}{2}\sigma^2 k^2} e^{-ikx}$$
$$= \frac{\omega c}{2(\omega - i\alpha c)} e^{-\frac{1}{2}\sigma^2((\omega/c)^2+\alpha^2)} e^{-i(\omega/c)x} e^{-\alpha x}$$

Now assuming that the weak attenuation in the $\alpha \ll \frac{\omega}{c}$layer we can eliminate the imaginary term from the above equation (Magrini, F., & Boschi, L. 2021), so that

$$v_z(x,\omega) \cong \frac{c}{2} e^{-\frac{1}{2}\sigma^2((\omega/c)^2+\alpha^2)} e^{-i(\omega/c)x} e^{-\alpha x}$$

With the Fourier inverse the equation above becomes

$$v_z(x,t) = \frac{c^2}{2\sigma\sqrt{2\pi}} e^{\frac{-(x-ct)^2}{2\sigma^2}} e^{\frac{-\alpha^2\sigma^2}{2\sigma^2}} e^{-\alpha x}$$

In the case of seismic waves as the waves travel through a medium, their intensity decreases with distance. In idealized materials, the amplitude of the waves only decreases due to wave propagation (Donà, M., Lombardo, M., & Barone, G. 2015). However, all natural materials produce effects that further attenuate the waves. This further attenuation is due to scattering and absorption. Scattering is the reflection of waves in directions other than their original direction of propagation. Absorption is the conversion of wave energy to another form of energy (Lutedx, T. R., 2016). Ideal attenuation is not something that is ruled out, but linear perturbation analysis aims to capture the mathematical analysis by assuming that the deeper perturbation terms can be very small and can be neglected. By applying the first order linear perturbation value, namely $c \equiv c_0 + c_1|\omega|$, $\alpha \equiv \alpha_0 + \alpha_1|\omega|$ where $c_0 \gg c_1\omega$ and $\alpha_0 \gg \alpha_1\omega$ and the infinite series expansion of the Macclaurin series is $\frac{1}{1+\frac{c_1}{c_0}|\omega|} \cong 1 - \frac{c_1}{c_0}|\omega|$. This very small disturbance does not change the basic system of wave propagation.

$$v_z(x,\omega) \cong \frac{c_0}{2} e^{-\frac{1}{2}\sigma^2(\omega/c_0)^2} e^{-i\left(\frac{\omega}{c_0+c_1|\omega|}\right)x} e^{-\alpha_1|\omega|x}$$

$$v_z(x,\omega) \cong \frac{c_0}{2} e^{-\frac{1}{2}\sigma^2\left(\frac{\omega}{c_0}\right)^2} e^{-ix\left(\frac{\omega}{c_0} - \frac{c_1\omega^2 sign\,|\omega|}{c_0{}^2}\right)} e^{-\alpha_1|\omega|x}$$

By using the relationship $v_z(x,\omega) = i\omega u_z(x,\omega)$ and gauss band width $\frac{c_1 sign\,|\omega|}{c_0{}^2} x = 1$ that

$$u_z(x,\omega) \cong \frac{-ic_0}{2\omega} e^{-\frac{1}{2}\sigma^2\left(\frac{\omega}{c_0}\right)^2} e^{-ix\left(\frac{\omega}{c_0} - \omega^2\right)} e^{-\alpha_1|\omega|x}$$

With α_1 is the perturbation absorption coefficient in the layer. From this equation we can see that the wave function in dispersive materials can change depending on the depth of the material and the magnitude of the linear absorption coefficient and the perturbation absorption coefficient. In the case of seismic waves, the waves are transmitted through an elastic layer so that the waves experience attenuation (Fang, L., & Leamy, M. J., 2024). The presence of disturbances in the layer through which the waves pass causes the material shift to experience a disturbance characteristic where the wave amplitude becomes smaller. This is because the absorption of wave energy by the particles of the dispersive medium continues to experience attenuation (Ridolfi, L., Torino, P., & Torino, P., 2015).

CONCLUSION

Wave dispersion is the process of changing the shape or energy of waves when passing through a dispersive medium. The process of changing waves is caused by the interaction of waves with the medium. This interaction can dampen the propagating waves. One aspect is seismic waves that propagate on the surface or depth of the layer. Additional aspects of the characteristics of the medium and the depth of the layer are the main factors causing the spread of the waves. In the case of seismic waves when the waves move through the medium, their intensity decreases with distance.

In idealized materials, the wave amplitude only decreases due to wave spreading. The quantity that states the comparative constant between the magnitude of the absorbed radiation intensity and the thickness of a material or material is called the attenuation coefficient (μ). However, all natural materials produce effects that further increase the waves. This further attenuation is caused by scattering and absorption. Scattering is the reflection of waves in a direction other than the original direction of propagation. Absorption is the conversion of wave energy to another form of energy. Ideal attenuation is not something that is set aside, but linear disturbance (attenuation) analysis aims to capture mathematical analysis by assuming that there is a disturbance in the layer through which the wave passes, causing the material shift to experience a disturbance characteristic where the wave amplitude is getting smaller. This is because the absorption of wave energy by the dispersed medium particles continues to experience attenuation so that the deeper disturbance terms can have very small values and can be ignored.

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